Open Source Software Subsidies and Network Compatibility in a Mixed Duopoly

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Abstract
For many applications, Open source software (OSS) can offer a high-quality alternative to proprietary software (e.g. Linux, Apache,...). But even if OSS is usually free of charge, its installation and use require some skills. Should the government intervene to promote the diffusion of OSS and provide some learning support to potential users? This paper analyses whether public subsidies towards open source software is socially desirable and how the extent of compatibility between open source software and proprietary software can influence the amount of subsidies. We consider a mixed duopoly model with a proprietary software (PS) company that competes with an open source software (OSS) community. An important strategic decision is whether to make these software compatible or incompatible. Four situations are possible: full (two-way) compatibility, full incompatibility, and one-way incompatibility (either only OSS or only PS is compatible). Users are heterogeneous in their ability to use OSS, and they are also sensitive to the number of users who are adopting the same software or a compatible software (network externality). We show that if the government only takes care of consumer surplus, public subsidies are welfare-enhancing. But the optimal subsidies are larger with full compatibility and PS compatibility than full incompatibility and OSS compatibility. These results suggest that government policy towards OSS must be conditional to the degree of compatibility between PS and OSS. However if the government is maximizing the total welfare (including the firm’s profit), subsidies towards open source are not socially desirable regardless of the regime of software compatibility.

1 Introduction
For many applications, Open source software (OSS)\(^1\) can offer a high-quality alternative to proprietary software (e.g. Linux, Apache, Gimp, Sendmail, etc.). They are usually

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\(^1\)OSS is software for which the source code is freely available, and that the license under which is distributed enables each user not only to use the software, but also to copy it, to modify it, and to redistribute the original or modified version to other users.
more reliable and can be customized to meet specific needs. But even if OSS are usually 
free of charge, its installation and use require some skills. Proprietary software tends to 
be more user-friendly and offers more help and support. A typical example is Linux, an 
open source alternative to the proprietary operating system Windows. Despite its superior 
quality, Linux has a limited market share and has mostly been adopted by expert users.² 
OSS advocates are pushing for government intervention to promote the diffusion of OSS 
(Varian and Shapiro, 2003; Benkler, 2002; Smith, 2002). Over the last decade, several 
(national and local) governments around the world have actively encouraged the adoption 
of OSS, through training programs and public procurement of OSS (for administrations 
and schools).

This article aims to examine whether public policy in favor of OSS can be efficient and 
how they impact users and proprietary software companies. Here we will focus on public 
subsidies or support directed to users to reduce their cost of OSS adoption (i.e. the cost 
of installing and using OSS): (i) What is the welfare impact of public subsidies for OSS? 
(ii) How the extent of compatibility between PS and OSS can influence the amount of 
subsidies?

The public policies towards open source software have yet been studied in the open 
source litterature. Schmidt and Schnitzer (2002) develop a model of spatial competition 
between PS and OSS in the presence of heterogeneous users. They discuss the welfare 
impact of public policies that encourage administrations, schools and universities to adopt 
OSS. They show that an increase in the open source market share reduces the proprietary 
software firm’s incentives to improve the quality of its software, which further reduces the 
total welfare. Mustonen (2003) shows that public efforts to provide better information 
on open source alternative are welfare increasing. Comino and Manenti (2003) examine 
subsidies for OSS users in a market where some potential users ignore the existence and/or 
characteristics of OSS. They show that OSS subsidies always reduce social welfare.

In this paper, we develop a model that extends the model Comino and Manenti (2003) 
by introducing a subjective cost or disutility of using open source software. We also assume 
that the proprietary and open source software are vertically differentiated (whereas in 
Comino and Manenti (2003), the two products have the same quality. Finally, we examine 
different regimes of software compatibility between PS and OSS.

More specifically, our model considers a firm that sells PS, an OSS community that 
offers OSS free of charge and a government that subsidizes users that adopt OSS. An 
important strategic decision is whether to make these software compatible or incompatible. 
Four situations are possible: full (two-way) compatibility, full incompatibility, and one-
way compatibility (either only OSS or only PS is compatible). OSS compatibility means 
that the PS users can unilaterally access the OSS community and derive benefits from 
it. For instance, they can use some programs or applications developed by the OSS 
community, or read and modify the files sent by the OSS users, whereas the OSS users 
cannot access the files or applications of the proprietary software. With PS compatibility, 
only OSS users can derive some utility from the PS users. We also assume that users are

²Network effects can also hinder the entry of higher quality software. Network effects arise both directly 
from the number of consumers who are using compatible software and indirectly from the provision 
of complementary services. Such network effects may tip the market in favor of only one software product. 
This can happen for any product or technology with network externality. For instance, it can explain the 
dominance of QWERTY keyboard even if it is less performant than the DSKs of August Dvorak (David, 
1985).
heterogeneous in their ability to use OSS, and they are also sensitive to the number of users who are adopting the same software or a compatible software.

We show that if the government only takes care of consumer surplus, public subsidies are welfare-enhancing. But the optimal subsidies are larger with full compatibility and OSS compatibility than full incompatibility and PS compatibility. These results suggest that government policy towards OSS must be conditional to the degree of compatibility between PS and OSS. However, if the government is maximizing the total welfare (including the firm’s profit), subsidies towards open source are not socially desirable regardless of the regime of software compatibility.

The paper is organized as follows. The next section presents the parameters of the model. Results of the two stage model are displayed in Section 3 and 4. In Section 5, we compare the price, profits, market shares and subsidies under the four compatibility regimes. Policy and managerial implications are discussed in Section 6.

2 Model

We consider a firm that sells a proprietary software (PS) of quality, $V_{PS}$, at price, $p$. But consumers have also the alternative to use an open source software (OSS) developed by an open source community. This software is free and has a level of quality, $V_{OS}$. We assume that $V_{OS} > V_{PS}$ meaning that the OSS has a superior quality (e.g. more features and better performance). This assumption is quite realistic. For instance, in the operating system market, Linux has less bugs and is updated more frequently than Windows (Raghunathan et al., 2005). However, OSS require some skills or expertise to be set up and used. We define $\Delta = V_{OS} - V_{PS} > 0$. In the remainder of the paper, we suppose that $V_{OS}$ and $V_{PS}$ are sufficiently large to ensure that the market is fully covered.

We assume that users are heterogeneous in their skills and bear a cost to use an OSS that directly depends on their level of expertise. Users’ skills $\theta$ are uniformly distributed on $(0, 1)$: for high skilled users, $\theta$ is closed to 0 and for low skilled users $\theta$ is closed to 1. For a given level of expertise $\theta$, the cost of using an OSS is equal to $c\theta$. The cost of using a proprietary software is assumed to be zero as PS are usually characterized by user-friendly interface. For simplicity, the mass of users is equal to 1 and users only adopt one software (no multi-adoptions).

User’s utility depends on the (intrinsic) quality of the software, but also on network effects. We suppose that users are identical in their valuation of software quality. Moreover, as Katz and Shapiro (1985) and Shy (2001), we assume that network effects are a linear function of the number of users who have adopted the same software or a compatible software. More users mean that it is easier to share or exchange data and files or get support. As the number of software users is only known after users make their adoption choice, they have to form expectations about the respective number of OSS and PS users. We suppose that each user correctly anticipates the size of each software network (self-fulfilling beliefs). The value of network externalities is $\gamma$ times the expected number

3Proprietary software is more user friendly than open source software because open source software is developed by high skilled programmers who are also the potential users of these software. For example, the installation of open source software require downloading source code, linking libraries, setting environment variables for the operating system and compelling the source code. In contrast, most proprietary software requires just a few clicks and technical support is usually available.
of users who have adopted the same software or a compatible software\(^4\).

We distinguish four situations, depending on whether the PS and the OSS are fully (two-way) compatible, partially (one-way) compatible or fully incompatible:

- **Full incompatibility**: if PS and OSS are fully incompatible, the value of network externality for PS users is \(\gamma N_{PS}\) (with \(N_{PS}\) the number of users who have adopted the PS) and the value of network externality for OSS users is \(\gamma N_{OS}\) (with \(N_{OS}\) the number of users who have adopted the OSS)

- **Full compatibility**: if PS and OSS are fully compatible, then the value of network externality for both users of OSS and PS is \(\gamma (N_{OS} + N_{PS}) = \gamma\) (as the market is fully covered and the total number of users is 1, we have \(N_{OS} + N_{PS} = 1\))

- **OSS-compatibility**: if OSS is unilaterally compatible, PS users can access the OSS community and derive utility from it, but OSS users cannot get any utility from the network of PS users. In this case, the value of network externality for PS users is \(\gamma (N_{PS} + N_{OS}) = \gamma\), and the value of network externality for OSS users is \(\gamma N_{OS}\).

- **PS-compatibility\(^5\)**: if PS is unilaterally compatible, only OSS users can access PS users and the value of network externality for OSS users and PS users is respectively \(\gamma (N_{PS} + N_{OS}) = \gamma\), and \(\gamma N_{PS}\).

For sake of simplicity, let II (resp. CC) denote the full incompatibility case (resp. full compatibility case). Similarly, let CI (resp. IC) denote the OSS-compatibility case (resp. PS-compatibility case). We define \(k = II, CC, CI, IC\).

The utility of type \(\theta\) user under the different compatibility scenarios is given by:

\[
U_{\theta} = V_{PS} + \gamma N_{PS} - p \quad \text{if the user buys a PS that is OS-incompatible (1.a)}
\]

\[
U_{\theta} = V_{OS} + \gamma N_{OS} - c\theta + s \quad \text{if the user downloads OS that is PS-incompatible (1.b)}
\]

\[
U_{\theta} = V_{PS} + \gamma - p \quad \text{if the user buys PS that is OS-compatible (1.c)}
\]

\[
U_{\theta} = V_{OS} + \gamma - c\theta + s \quad \text{if the user downloads OS that is PS-compatible (1.d)}
\]

Assuming that the marginal cost to sell a PS is constant and normalized to zero, the profit of the software firm is given by:

\[
\Pi^k = p^k N_{PS}^k \quad \text{with } k = II, CC, CI, IC \quad (2)
\]

\(^4\)Following Farrell and Saloner (1992), we assume that the value of network externality \(\gamma\) is the same for both software.

\(^5\)This case is less realistic as unilateral compatibility from PS to OSS is seldom observed.
By definition the OS community has no revenues and no cost (i.e. profit equal to zero)\textsuperscript{6}.

In this paper, we analyze the impact of subsidies that are directed to OSS users. These subsidies can take the form of training sessions, support,... to reduce the cost of adoption of OSS. Let \( s \) be the amount of subsidies per user with \( s > 0 \). What should be the optimal level of subsidies or support by the government?

Let \( S = sN_{\text{OS}} \) be the cost of subsidizing OSS users. The social welfare is the sum of firm’s profit (\( \Pi \)) and consumers’ surplus (\( CS \)) minus the cost of subsidies:

\[
W^k = \Pi^k + CS^k - S^k \quad \text{with} \quad k = \text{II, CC, CI, IC} \quad (3)
\]

In our paper, we will consider that the government only takes into account the consumers’ surplus. This can be justified by two reasons. Firstly, many software companies are operating abroad and their profits cannot be part of the domestic social surplus\textsuperscript{7}. Secondly in matters of market regulation and competition policy, governments tend to put more weight on consumers’ welfare.

In this case, the government will choose the amount of subsidies that maximizes the following function:

\[
W^k_U = US^k - S^k \quad \text{with} \quad k = \text{II, CC, IC, IC} \quad (4)
\]

In the Appendix, we also present the results when the government maximizes the social welfare that includes the profit of the software firm (see (3)).

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\text{OS}}, V_{PS} )</td>
<td>The intrinsic quality of OSS and PS</td>
</tr>
<tr>
<td>( p )</td>
<td>Price set by the software firm</td>
</tr>
<tr>
<td>( c )</td>
<td>Cost of using OSS</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Level of expertise</td>
</tr>
<tr>
<td>( s )</td>
<td>Subsidy per OSS user</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Valuation of network externality</td>
</tr>
<tr>
<td>( N_{\text{OS}}, N_{PS} )</td>
<td>Market share of OSS and PS</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>Profit of firm</td>
</tr>
<tr>
<td>( S )</td>
<td>Tax burden of subsidizing OSS</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>Difference in the qualities of OSS and PS</td>
</tr>
</tbody>
</table>

The timing of the strategic game is as follows. In the first stage, the government chooses the amount of the subsidies to help uses to adopt the OSS. In the second stage, the firm sets the price of its software and then the users choose to adopt either the PS or the OSS. In the following section, we start to solve the second stage of the model under the four compatibility scenarii.

\textsuperscript{6}Open source software are developped by open source communities whose members voluntarily contribute . Since open source software is freely downloadable on the Internet, these communities do not earn a profit.

\textsuperscript{7}The main software companies are US companies and this explains why many European government, but also China, want to encourage the adoption of OSS (especially for public administrations and schools).
3 Price decision and market shares

3.1 Full incompatibility

When the software products offered by the firm and the OS community are fully incompatible, network externalities for users of OSS and PS users are respectively $\gamma N_{OS}$, and $\gamma N_{PS}$. Then the utility of type $\theta$ user is given by

$$U_{\theta}^{II} = \begin{cases} V_{PS} + \gamma N_{PS} - p & \text{if choice of PS} \\ V_{OS} + \gamma N_{OS} - c\theta + s & \text{if choice of OSS} \end{cases}$$

Let $\hat{\theta}$ be the marginal user who is indifferent between adopting PS and OSS. Since the users are uniformly distributed between $(0, 1)$, then $N_{OS} = \hat{\theta}$ and $N_{PS} = 1 - \hat{\theta}$. Solving $V_{PS} + \gamma(1 - \hat{\theta}) - p = V_{OS} + \gamma\hat{\theta} - c\hat{\theta} + s$ yields $\hat{\theta} = \frac{(p+s-\gamma+\Delta)}{c-2\gamma}$. Users with a type $\theta < \hat{\theta}$ (high skilled) will prefer OSS and users with $\theta > \hat{\theta}$ will adopt PS.. It implies that market shares can be expressed as:

$$N_{OS}^{II} = \frac{p + s - \gamma + \Delta}{c - 2\gamma}$$

$$N_{PS}^{II} = \frac{c - p - s - \gamma - \Delta}{c - 2\gamma}$$

To ensure that PS and OSS have positive market shares at the equilibrium (i.e. to eliminate corner solution) we need to assume that $c - 2\gamma > 0$ and that $\Delta < c - 2\gamma$ (see appendix). In other terms, the cost of OSS adoption must be sufficiently high or network externalities not too large. Under these two conditions and from (2), the firm’s profit is

$$\Pi^{II} = pN_{PS}^{II} \iff p \left( \frac{c - p - s - \gamma - \Delta}{c - 2\gamma} \right)$$

Given the amount of subsidies $s$, the firm chooses the profit-maximizing price that is equal to

$$p^{II}(s) = \frac{c - s - \gamma - \Delta}{2}$$

We observe that the price decreases with the amount of subsidies. The effect of subsidies is to increase competition between the two types of software and reduce the market power of the software firm.

After rearrangement the market shares are

$$N_{OS}^{II}(s) = \frac{c + s - 3\gamma + \Delta}{2(c - 2\gamma)}$$

$$N_{PS}^{II}(s) = \frac{c - s - \gamma - \Delta}{2(c - 2\gamma)}$$

and the equilibrium profit is

If network externalities are too strong, i.e. $c < 2\gamma$, the firm exits the market and the OSS is the only software adopted by users.
\[ \Pi''(s) = \frac{(c-s-\gamma-\Delta)^2}{4(c-2\gamma)} \] (5.7)

Note that the firm and the OS community have a positive market share if and only if \( c - s - \gamma > \Delta > 3\gamma - c - s. \)

### 3.2 Full (Two-way) Compatibility

When the software products are two-way compatible, any user is able to interact with both PS and OSS users. The value of network externalities is \( \gamma (N_{OS} + N_{PS}) = \gamma \) whatever the software adopted and the utility of a type \( \theta \) is

\[
U^\theta_{CC} \equiv \begin{cases} 
V_{PS} + \gamma - p & \text{if choice of PS} \\
V_{OS} + \gamma - c\theta + s & \text{if choice of OSS}
\end{cases}
\]

The user indifferent between the two software products is characterized by \( \hat{\theta} = \frac{(p+s+\Delta)}{c} \) and the market shares are respectively:

\[
N^CC_{OS} = \frac{p + s + \Delta}{c} \] (6.1)

\[
N^CC_{PS} = \frac{c - p - s - \Delta}{c} \] (6.2)

As the firm’s profit is

\[
\Pi^CC = pN^CC \iff p \left( \frac{c - p - s - \Delta}{c} \right) \] (6.3)

the optimal price is given by

\[
p^{CC}(s) = \frac{c - s - \Delta}{2} \] (6.4)

As previously, the price of PS decreases with subsidies Then the equilibrium market shares and profit can be rewritten as follows

\[
N^CC_{OS}(s) = \frac{c + s + \Delta}{2c} \] (6.5)

\[
N^CC_{PS}(s) = \frac{c - s - \Delta}{2c} \] (6.6)

\[
\Pi^{CC}(s) = \frac{(c - s - \Delta)^2}{4c} \] (6.7)

Note that the firm and the OS community have a positive market share if and only if \( c > \Delta + s. \) Otherwise only the OS community will remain active in the market.
3.3 OSS (one-way) compatibility

If the compatibility is only from OSS toward PS, then the PS users can use programs that are developed by the OS community and enjoy more network externality. Then the utility of a type $\theta$ is

$$U^{CI}_\theta \equiv \begin{cases} V_{PS} + \gamma - p & \text{if choice of PS} \\ V_{OS} + \gamma N_{OS} - c\theta + s & \text{if choice of OSS} \end{cases}$$

The user who is indifferent between adopting the OSS and the PS product is characterized by $\hat{\theta} = \frac{p + s - \gamma + \Delta}{c - \gamma}$ assuming that $c > \gamma$. Then, the market shares are given by:

$$N^{CI}_{OS} = \frac{(p + s - \gamma + \Delta)}{c - \gamma}$$

(7.1)

$$N^{CI}_{PS} = \frac{(-p + c - s - \Delta)}{c - \gamma}$$

(7.2)

The optimal price for the proprietary software is $p^{CI}(s) = \frac{c - s - \Delta}{2}$, and the equilibrium market shares and profit are as follows

$$N^{CI}_{OS}(s) = \frac{(c + s - 2\gamma + \Delta)}{2(c - \gamma)}$$

(7.5)

$$N^{CI}_{PS}(s) = \frac{c - s - \Delta}{2(c - \gamma)}$$

(7.6)

$$\Pi^{CI}(s) = \frac{(c - s - \Delta)^2}{4(c - \gamma)}$$

(7.7)

Note that the Firm and the OS community have a positive market share if and only if $c - s > \Delta > 2\gamma - c - s$.

3.4 PS (one-way) compatibility

The last (but probably less realistic) scenario is a PS compatibility regime. OSS users can access the network of PS users and derive some utility from it. The utility of type $\theta$ user is

$$U^{IC}_\theta \equiv \begin{cases} V_{PS} + \gamma N_{PS} - p & \text{if choice of PS} \\ V_{OS} + \gamma - c\theta + s & \text{if choice of OSS} \end{cases}$$

The user who is indifferent between adopting the OSS and the PS product is characterized by $\hat{\theta} = \frac{(p + s + \Delta)}{c - \gamma}$ (assuming $c > \gamma$). Then, the market shares are given by

$$N^{IC}_{OS} = \frac{p + s + \Delta}{c - \gamma}$$

(8.1)

$$N^{IC}_{PS} = \frac{c - p - s - \gamma - \Delta}{c - \gamma}$$

(8.2)

The profit-maximizing price for the proprietary software is $p^{IC}(s) = \frac{c - s - \gamma - \Delta}{2}$, and the equilibrium market shares and profit are as follows...
\[ N_{OS}^{IC}(s) = \frac{c + s - \gamma + \Delta}{2(c - \gamma)} \]  
\[ N_{PS}^{IC}(s) = \frac{c - s - \gamma - \Delta}{2(c - \gamma)} \]  
\[ \Pi^{IC}(s) = \frac{(c - s - \gamma - \Delta)^2}{4(c - \gamma)} \]  

Note that the firm and the OS community have a positive market share if and only if \( c - s - \gamma > \Delta > \gamma - c - s \).

### 4 Open source subsidy decision of the government

Now, we consider the first-stage of the model where the government sets the subsidy per OSS users in order to maximize the consumers’ surplus (net of the cost of subsidies). Solving the first order condition under the four scenarii, the optimal subsidies are given by

\[ s^{CC*} = \frac{c - \Delta}{3} \quad ; \quad s^{CI*} = \frac{c - \Delta}{3} \quad ; \quad s^{IC*} = \frac{c - \gamma - \Delta}{3} \quad ; \quad s^{II*} = \frac{c - \gamma - \Delta}{3} \]  

By replacing \( s \) by its optimal value, we can derive the equilibrium prices, market shares (or the users base for each type of software) and the profit that are summarized in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>II</th>
<th>CC</th>
<th>CI</th>
<th>IC</th>
</tr>
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<tbody>
<tr>
<td>( p^* )</td>
<td>( \frac{c - \gamma - \Delta}{3} )</td>
<td>( \frac{c - \Delta}{3} )</td>
<td>( \frac{c - \Delta}{3} )</td>
<td>( \frac{c - \gamma - \Delta}{3} )</td>
</tr>
<tr>
<td>( s^* )</td>
<td>( \frac{c - \Delta}{3} )</td>
<td>( \frac{c - \Delta}{3} )</td>
<td>( \frac{c - \gamma - \Delta}{3} )</td>
<td>( \frac{c - \gamma - \Delta}{3} )</td>
</tr>
<tr>
<td>( N_{PS}^{*} )</td>
<td>( \frac{c - \gamma - \Delta}{3(c - \gamma)} )</td>
<td>( \frac{c - \Delta}{3(c - \gamma)} )</td>
<td>( \frac{c - \Delta}{3(c - \gamma)} )</td>
<td>( \frac{c - \gamma - \Delta}{3(c - \gamma)} )</td>
</tr>
<tr>
<td>( N_{OS}^{*} )</td>
<td>( \frac{2c - 5\gamma + \Delta}{3(c - \gamma)} )</td>
<td>( \frac{2c - \Delta}{3(c - \gamma)} )</td>
<td>( \frac{2c - 3\gamma + \Delta}{3(c - \gamma)} )</td>
<td>( \frac{2c - 2\gamma + \Delta}{3(c - \gamma)} )</td>
</tr>
<tr>
<td>( \Pi^{*} )</td>
<td>( \frac{(c - \gamma - \Delta)^2}{9(c - \gamma)} )</td>
<td>( \frac{(c - \Delta)^2}{9(c - \gamma)} )</td>
<td>( \frac{(c - \gamma - \Delta)^2}{9(c - \gamma)} )</td>
<td>( \frac{(c - \gamma - \Delta)^2}{9(c - \gamma)} )</td>
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### 5 Implications for public policy

In this section, we conduct some comparative statics analysis on the equilibrium outcomes across the four compatibility regimes. These results provides some insights on the desirability of public efforts to promote OSS. To allow comparisons across the four scenarii, we restrict our analysis to the situations where PS and OSS have both positive market shares whatever the regime of compatibility. The necessary conditions to have coexistence of PS and OSS are \( c > \Delta + \gamma \) and \( \Delta > \gamma \).

\footnote{In each regime of compatibility, the optimal subsidy is unique as \( \frac{\partial^2 W^s}{\partial s^2} < 0 \) for \( c > \Delta + \gamma \) and \( \Delta > \gamma \).}
5.1 Comparison of subsidies, public deficits, market shares, prices and profits

The next proposition compares the optimal subsidy under the four compatibility regimes:

**Proposition 1** Assuming that a sub-game perfect equilibrium exists where both firm and OS community are active in the market, i.e. implementation cost of the OSS is sufficiently higher with \( c > \Delta + \gamma \) and \( \Delta > \gamma \). Then, \( s_{CC}^* = s_{CI}^* > s_{IC}^* = s_{II}^* \)

**Proof.** See Table 2.

The government gives a larger subsidy per user in the cases of two-way compatibility and OSS compatibility. This result is quite intuitive. When the OSS is compatible with the PS, then PS users derive some gain from an additional OSS user (through network externality). This is not the case under full incompatibility and PS compatibility. Clearly the government has more incentives to subsidize OSS users in the CC and CI situation as the return in terms of welfare will be larger.

Proposition 2 compares the PS price, market shares, and profit under the four compatibility situations.

**Proposition 2** When the government gives welfare-maximizing subsidies to OSS users, then :

\[
\text{(i)} \quad p_{CC}^*|_{s_{CC}^*} = p_{CI}^*|_{s_{CI}^*} = p_{IC}^*|_{s_{IC}^*} = p_{II}^*|_{s_{II}^*} ;
\]

\[
\text{(ii)} \quad N_{PS}^*|_{s_{II}^*} > N_{PS}^*|_{s_{CI}^*} > N_{PS}^*|_{s_{CC}^*} > N_{PS}^*|_{s_{IC}^*} ;
\]

\[
\text{(ii)} \quad \Pi_{II}^*|_{s_{II}^*} > \Pi_{CI}^*|_{s_{CI}^*} > \Pi_{CC}^*|_{s_{CC}^*} > \Pi_{IC}^*|_{s_{IC}^*} .
\]

**Proof.** See Table 2 and Appendix 3.

The firm sets a higher price when the utility of its customers increases with the number of OSS users. As they are willing to pay more for the proprietary software, the firm is able to increase its price in the situations of full compatibility or OSS compatibility. However, the ranking of the four scenario in terms of price and profit is different. The firm is better under full incompatibility: it enjoys a larger market share and profit. The second best situation is OSS compatibility in which users are charged a higher price for the PS and receive more subsidies for the OSS compared to the full incompatibility case. The results is a lower market share for the proprietary software and profit than under full incompatibility. The worst situation for the software firm (in terms of market share and profit) is PS compatibility because users get more utility to adopt OSS. The firm has to be more aggressive in its pricing but it’s not sufficient to retain its consumers given the subsidies distributed by the government. The situation of full compatibility is between the OSS and PS compatibility cases. The price of the PS (and the subsidy per OSS user) under full compatibility is the same as under OSS compatibility, but its market share is lower because the firm has no exclusive advantage in terms of network externality under full compatibility. Its product is less attractive than under PS compatibility. If the firm has the possibility to choose the compatibility regime, it is incited to deny access to its services and customer base (by making its software incompatible for OSS users).

Proposition 3 compares the fiscal burden of these subsidies.
Proposition 3 Assuming that a sub-game perfect equilibrium exists where both firm and OS community are active in the market, i.e. implementation cost of the OSS is sufficiently higher with \( c > \Delta + \gamma \) and \( \Delta > \gamma \). Then \( S_{CC} > S_{CI} > S_{IC} > S_{II} \)

Proof. See Appendix 4. 

An open source software policy is more costly when the PS and OSS are fully compatible because the number of subsidy beneficiaries is higher under full compatibility than under OSS compatibility even if the subsidy per user is the same in both situations. The full incompatibility situation is the least costly for the government as the subsidy per user is lower than in the CC and CI regimes and the number of beneficiaries is also lower.

5.2 Comparative statics across software compatibility situations

We also analyze how the equilibrium outcome change when the cost of OSS adoption \( (c) \), the gap between OSS and PS quality \( (\Delta) \), and the magnitude of network externality \( (\gamma) \) vary.

<table>
<thead>
<tr>
<th></th>
<th>( CC )</th>
<th>( \Delta )</th>
<th>( \gamma )</th>
<th>( II )</th>
<th>( \Delta )</th>
<th>( \gamma )</th>
<th>( CI )</th>
<th>( \Delta )</th>
<th>( \gamma )</th>
<th>( IC )</th>
<th>( \Delta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{CC} )</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>( s_{II} )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>( s_{CI} )</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>( s_{IC} )</td>
</tr>
<tr>
<td>( p_{CC} )</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>( p_{II} )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>( p_{CI} )</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>( p_{IC} )</td>
</tr>
<tr>
<td>( N_{PS} )</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>( N_{PS} )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>( N_{PS} )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>( N_{PS} )</td>
</tr>
<tr>
<td>( N_{OS} )</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>( N_{OS} )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>( N_{OS} )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>( N_{OS} )</td>
</tr>
<tr>
<td>( \Pi_{CC} )</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>( \Pi_{II} )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>( \Pi_{CI} )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>( \Pi_{IC} )</td>
</tr>
</tbody>
</table>

Table 3: Impact of \( c \) (cost of OSS adoption), \( \Delta \) (quality advantage of OSS) and \( \gamma \) (magnitude of network externality) on subsidies, price, market shares and profits under Full compatibility (CC), Full incompatibility (II), OSS-compatibility (CI) and PS-compatibility (IC). Interpretation: + means positive impact, - means negative impact and 0 means neutral effect.

Table 3 shows that the firm’s profit, prices and market shares, as well as the subsidies, increase with the cost of using OSS. Moreover, the subsidies and the price charged by the firm, as well as its profit, tend to decrease with the differential in quality between the OSS and the PSS.

However, the role of network externality differs under the four regimes. It has no impact on market shares and profits under full compatibility, but it has a positive impact on firm’s market share under full incompatibility and OSS compatibility and a negative impact under PS compatibility. Moreover, when consumers have a high valuation for network externality, the firm is better off when its consumers can unilaterally benefit from OSS’s users base. When it’s not the case, an increase in the value of network externality will induce the firm to price more aggressively, and its profit will be reduced.
5.3 Comparison of Welfare Levels

Finally, we compare social welfare (excluding firm’s profit) under the four compatibility situations. Welfare under full compatibility, full incompatibility, OSS compatibility and PS compatibility are, respectively:

\[ W_{CC}^* = \frac{(4cV_{OS} + 2cV_{PS} + 6\gamma c + \Delta^2 - 2c^2)}{6c}; \]  
\[ W_{II}^s = \frac{(7c\gamma - 5\gamma^2 + 4cV_{OS} + 2cV_{PS} - 10\gamma V_{OS} - 2\gamma V_{PS} - 2c^2 + \Delta^2)}{6(c - 2\gamma)}; \]  
\[ W_{CI}^* = \frac{(6c\gamma + 4cV_{OS} + 2cV_{PS} + \Delta^2 - 3\gamma^2 - 6\gamma V_{OS} - 2c^2)}{6(c - \gamma)}; \]  
\[ W_{IC}^* = \frac{(7c\gamma + 4cV_{OS} + 2cV_{PS} + \Delta^2 - 5\gamma^2 - 4\gamma V_{OS} - 2\gamma V_{PS} - 2c^2)}{6(c - \gamma)}. \]

**Proposition 4** When the government gives welfare-maximizing subsidies to OSS users, then:

\[ W_k|_{s=s^*} - W_k|_{s=0} > 0 \quad \text{for } k=II, \ CI, \ CC, \ IC \]

and

\[ W_{CC}^*|_{s=s^*} > W_{IC}^*|_{s=s^*} > W_{CI}^*|_{s=s^*} > W_{II}^s|_{s=s^*}. \]

**Proof.** See Appendix 6.

The intervention of the government in favor of OSS is always welfare enhancing. But the welfare with subsidies is larger when the proprietary and OSS are perfectly compatible. In this situation, the welfare impact of subsidies is to stimulate competition and push the firm to reduce its price (but subsidies have no impact on network externality). The worst situation in terms of welfare is full incompatibility. Even if subsidies intensify competition, consumers’ surplus is lower as network externalities are only intra-network. The intermediate situations are one way compatibility regime with PS compatibility out-ranking OSS compatibility. When the OSS users benefit from unilateral network effects (PS compatibility), consumers are more likely to adopt the OSS that offers superior quality and extended network externality. Subsidies can reinforce this trend and increase the market share of OSS and the utility of OSS users.

In the Appendix B, we have examined the situation in which the government takes into account the firm’s profit. When the government objective is the sum of consumers’ surplus and firm’s profit, then public subsidies for OSS’s users have negative impact on welfare regardless of the compatibility regime and the best policy is laissez-faire or “public neutrality”. This finding is similar to the results obtained by Comino and Manenti (2003) with horizontal product differentiation between PS and OSS.

Our final proposition analyzes how a change in the cost of adopting OSS can impact the welfare gain of subsidies.
Proposition 5  \( \frac{\partial (W^k|_{x=c} - W^k|_{x=0})}{\partial c} > 0 \) for all \( c > \Delta + \gamma \) and \( \Delta > \gamma \).

Proof. See Appendix 8. ■

The difference in social welfare with and without subsidies increases with the (subjective) cost of using OSS regardless of the compatibility regime. It implies that subsidies are more efficient and more justified when there are a lot of obstacles and costs to adopt OSS.

6 Concluding remarks

Although the open source literature has extensively studied the issue of competition between open source and proprietary software, little attention has been paid to the impact of public policy to promote open source software in this context. The aim of this paper is to extend the literature. We analyze the impact of public subsidies for OSS users in presence of network effects and under different compatibility regimes.

Our main findings are that public subsidies push down the price of proprietary software, increase the market share of the OS software and may stimulate network externality when PS and OSS are partially incompatible (PS one-way compatibility). When only users’ welfare is taken into account and the adoption cost of OSS is sufficient high, public subsidies for OSS’ users are welfare-enhancing. However, the optimal policy is to provide larger subsidies per user under full compatibility and OSS one-way compatibility than under full incompatibility and PS one-way compatibility.

We have also examined the optimal policy when the government maximizes the total welfare (including the firm’s profit). In this case, subsidizing OSS is not socially desirable regardless of the regime of software compatibility. This result is similar to that obtained by Comino and Manenti (2003) with a model of horizontal differentiation and can be used as an argument in favor of a “technology neutrality” (meaning that a government should never intervene to sponsor a technology, but let the market choose the best technologies).

The theoretical model developed in our paper has several limitations. First, we consider that the quality of OSS and PS is exogenous. It would be interesting to add a stage in which the OS community and the software company can invest in the quality of their software. Moreover, the choice of compatibility could also be endogenized. Our results suggest the software company has strong incentives to make its product incompatible with the OSS. Another limitation is that our model is static and does not allow for intertemporal pricing strategies. Instead we could consider two periods and two generations of potential users. In the initial period, the software company could be more aggressive to get a critical mass of users and obtain competitive advantage (through network externality) in the second period. In this dynamic setting, optimal public subsidies could clearly be different over time.

In addition to these results, we have shown that the optimal OSS subsidy is higher under full compatibility and OSS compatibility followed by PS compatibility and full incompatibility.
References


Appendix A

A.1 Equilibrium values under the four compatibility situations

In this Appendix we present all the equilibrium values under the four compatibility situations when the government only cares about the surplus of users.

<table>
<thead>
<tr>
<th></th>
<th>II</th>
<th>CC</th>
<th>CI</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*$</td>
<td>$\frac{c^-\gamma - \Delta}{3}$</td>
<td>$\frac{c^-\Delta}{3}$</td>
<td>$\frac{c^-\Delta}{3}$</td>
<td>$\frac{c^-\gamma - \Delta}{3}$</td>
</tr>
<tr>
<td>$s^*$</td>
<td>$\frac{c^-\gamma - \Delta}{3}$</td>
<td>$\frac{c^-\Delta}{3}$</td>
<td>$\frac{c^-\Delta}{3}$</td>
<td>$\frac{c^-\gamma - \Delta}{3}$</td>
</tr>
<tr>
<td>$N_{PS}^*$</td>
<td>$\frac{c^-\gamma - \Delta}{3(c-2\gamma)}$</td>
<td>$\frac{c^-\Delta}{3c}$</td>
<td>$\frac{c^-\Delta}{3c}$</td>
<td>$\frac{c^-\gamma - \Delta}{3(c-\gamma)}$</td>
</tr>
<tr>
<td>$N_{OS}^*$</td>
<td>$\frac{2c^-\gamma + \Delta}{3(c-2\gamma)}$</td>
<td>$\frac{2c^-\Delta}{3c}$</td>
<td>$\frac{2c^-\Delta}{3c}$</td>
<td>$\frac{2c^-\gamma + \Delta}{3(c-\gamma)}$</td>
</tr>
<tr>
<td>$\Pi^*$</td>
<td>$\frac{(c^-\gamma - \Delta)^2}{9(c-2\gamma)}$</td>
<td>$\frac{(c^-\Delta)^2}{9c}$</td>
<td>$\frac{(c^-\Delta)^2}{9c}$</td>
<td>$\frac{(c^-\gamma - \Delta)^2}{9(c-\gamma)}$</td>
</tr>
</tbody>
</table>

Table 4: n.c. necessary condition for subgame-perfect equilibrium, differences between OSS and PS quality $\Delta = V_{OS} - V_{PS}$, Full compatibility (CC), Full incompatibility (II), OSS- compatibility (CI) and PS-compatibility (IC)

A.2. Necessary conditions for the coexistence of OSS and PS

A duopoly (with positive market shares for OSS and PS) exists under the four compatibility regimes if the following conditions are satisfied

$$0 < \theta^k < 1 \quad \text{and} \quad U_{\theta}^k > 0 \quad \text{with} \quad k = CC, II, CI, IC$$  \hspace{1cm} (A)

The first condition ensures an interior solution for the indifferent user between OSS and PS. The second condition ensures that the indifferent user gets a non-negative utility. Assume $U_{\theta}^k > 0$ holds. The first condition under full compatibility, full incompatibility, PS compatibility and OSS compatibility implies respectively:

$$0 < \theta^{CC} < 1 \implies 0 < \Delta < c$$  \hspace{1cm} (A.1)

$$0 < \theta^{II} < 1 \implies c > \max \left[ \frac{5\gamma - \Delta}{2}, 2\gamma, \Delta + \gamma \right]$$  \hspace{1cm} (A.2)

$$0 < \theta^{CI} < 1 \implies c > \max \left[ \frac{3\gamma - \Delta}{2}, \gamma, \Delta \right]$$  \hspace{1cm} (A.3)
0 < \theta^{IC*} < 1 \implies c > \max \left[ \frac{2\gamma - \Delta}{2}, \gamma, \Delta + \gamma \right] \quad (A.4)

Conditions (A.1) – (A.4) are equivalent to

0 < \theta^{k*} < 1 \implies c > \Delta + \gamma = c^d \text{ and } \Delta > \gamma \quad (A.5)

Condition (A.5) is sufficient to ensure the existence of the mixed duopoly under the four compatibility regimes.

A.3. Proof of proposition 2

- **Optimal prices**

As \( p_{PS}^{CC*} = \frac{c - \Delta}{3c} \); \( p_{PS}^{CI*} = \frac{c - \gamma - \Delta}{3c} \); \( p_{PS}^{IC*} = \frac{c - \gamma - \Delta}{3c} \); \( p_{PS}^{II*} = \frac{c - \gamma - \Delta}{3c} \), it implies that \( p_{PS}^{CC*} = p_{PS}^{CI*} > p_{PS}^{IC*} = p_{PS}^{II*} \).

- **Optimal market shares**

As \( N_{PS}^{CC*} = \frac{c - \Delta}{3c} \); \( N_{PS}^{CI*} = \frac{c - \Delta}{3(c - \gamma)} \); \( N_{PS}^{IC*} = \frac{c - \gamma - \Delta}{3(c - \gamma)} \); \( N_{PS}^{II*} = \frac{c - \gamma - \Delta}{3(c - 2\gamma)} \), and since \( c > \Delta + \gamma = c^d \text{ and } \Delta > \gamma \), then it implies that \( N_{PS}^{II*} > N_{PS}^{CI*} > N_{PS}^{CC*} > N_{PS}^{IC*} \).

- **Optimal profits**

As \( \Pi^{CC*} = \frac{(c - \Delta)^2}{9c} \); \( \Pi^{CI*} = \frac{(c - \Delta)^2}{9(c - \gamma)} \); \( \Pi^{IC*} = \frac{(c - \gamma - \Delta)^2}{9(c - \gamma)} \); \( \Pi^{II*} = \frac{(c - \gamma - \Delta)^2}{9(c - 2\gamma)} \), then it implies that \( \Pi^{II*} > \Pi^{CI*} > \Pi^{CC*} > \Pi^{IC*} \) (assuming that \( c > \Delta + \gamma \text{ and } \Delta > \gamma \)).

A.4. Proof of proposition 3

As \( S^{CC*} = \frac{(c - \Delta)(2c + \Delta)}{9c} \); \( S^{CI*} = \frac{(c - \Delta)(2c - 3\gamma + \Delta)}{9(c - \gamma)} \); \( S^{IC*} = \frac{(c - \gamma - \Delta)(2c - 2\gamma + \Delta)}{9(c - \gamma)} \); \( S^{II*} = \frac{(c - \gamma - \Delta)(2c - 5\gamma + \Delta)}{9(c - 2\gamma)} \), it can be shown that \( S^{CC*} > S^{CI*} > S^{IC*} > S^{II*} \) (assuming \( c > \Delta + \gamma \)) because

\[
S^{CC*} - S^{CI*} = \frac{1}{9c} \frac{(c - \Delta)^2}{c - \gamma} > 0 
\]

\[
S^{CI*} - S^{IC*} = \frac{1}{9c} \gamma (c + 2\Delta - 2\gamma) > 0 
\]

\[
S^{IC*} - S^{II*} = \frac{1}{9c} \gamma (\Delta - c + \gamma)^2 > 0 
\]

A.5. Proofs of Table 3

Comparative statics on price, market shares and profit of the firm:\(^{10}\)

\(^{10}\)Since values of price and subsidy under the all compatibility regimes are equal at the equilibrium, the verification of the comparative statics of the subsidy is ignored in this Appendix.
• Impact of \( c, \Delta \) and \( \gamma \) on price

\[
\frac{\partial p_{CCs}}{\partial c} = \frac{1}{3} > 0; \quad \frac{\partial p_{CCs}}{\partial \Delta} = -\frac{1}{3} < 0
\]

\[
\frac{\partial \Pi_{Is}}{\partial c} = \frac{1}{3} > 0; \quad \frac{\partial \Pi_{Is}}{\partial \Delta} = -\frac{1}{3} < 0; \quad \frac{\partial \Pi_{Is}}{\partial \gamma} = -\frac{1}{3} < 0
\]

\[
\frac{\partial p_{Cl}s}{\partial c} = \frac{1}{3} > 0; \quad \frac{\partial p_{Cl}s}{\partial \Delta} = -\frac{1}{3} < 0; \quad \frac{\partial p_{Cl}s}{\partial \gamma} = -\frac{1}{3} < 0
\]

• Impact of \( c, \Delta \) and \( \gamma \) on market share of the firm

\[
\frac{\partial N_{PS}^{CCs}}{\partial c} = \frac{1}{3c^2} \Delta > 0; \quad \frac{\partial N_{PS}^{CCs}}{\partial \Delta} = -\frac{1}{3 < 0}
\]

\[
\frac{\partial N_{PS}^{CI}s}{\partial c} = \frac{\Delta}{3(c-\gamma)^2} > 0; \quad \frac{\partial N_{PS}^{CI}s}{\partial \Delta} = -\frac{1}{3(c-\gamma)} < 0; \quad \frac{\partial N_{PS}^{CI}s}{\partial \gamma} = \frac{c-\Delta}{3(c-\gamma)^2} > 0
\]

\[
\frac{\partial N_{PS}^{ICs}}{\partial c} = \frac{\Delta}{3(c-\gamma)^2} > 0; \quad \frac{\partial N_{PS}^{ICs}}{\partial \Delta} = -\frac{1}{3(c-\gamma)} < 0; \quad \frac{\partial N_{PS}^{ICs}}{\partial \gamma} = -\frac{\Delta}{3(c-\gamma)^2} < 0
\]

\[
\frac{\partial N_{PS}^{IIs}}{\partial c} = \frac{1}{3(c-2\gamma)^2} > 0; \quad \frac{\partial N_{PS}^{IIs}}{\partial \Delta} = -\frac{1}{3(c-2\gamma)} < 0; \quad \frac{\partial N_{PS}^{IIs}}{\partial \gamma} = -\frac{c-2\Delta}{3(c-2\gamma)^2} > 0
\]

iff \( c > 2\Delta \) where \( c > c^d = \Delta + \gamma \). Thus, the duopoly condition is satisfied.

• Impact of \( c, \Delta \) and \( \gamma \) on market share of the OS community

\[
\frac{\partial N_{OS}^{CCs}}{\partial c} = -\frac{(\Delta-\gamma)}{3c^2} \Delta < 0; \quad \frac{\partial N_{OS}^{CCs}}{\partial \Delta} = \frac{1}{3c} > 0
\]

\[
\frac{\partial N_{OS}^{CI}s}{\partial c} = \frac{(\gamma-\Delta)}{3(c-\gamma)^2} < 0; \quad \frac{\partial N_{OS}^{CI}s}{\partial \Delta} = \frac{1}{3(c-\gamma)} > 0; \quad \frac{\partial N_{OS}^{CI}s}{\partial \gamma} = -\frac{c-\Delta}{3(c-\gamma)^2} < 0
\]

\[
\frac{\partial N_{OS}^{ICs}}{\partial c} = -\frac{\Delta}{3(c-\gamma)^2} < 0; \quad \frac{\partial N_{OS}^{ICs}}{\partial \Delta} = \frac{1}{3(c-\gamma)} > 0; \quad \frac{\partial N_{OS}^{ICs}}{\partial \gamma} = \frac{\Delta}{3(c-\gamma)^2} > 0
\]

\[
\frac{\partial N_{OS}^{IIs}}{\partial c} = -\frac{(\Delta-\gamma)}{3(c-2\gamma)^2} < 0; \quad \frac{\partial N_{OS}^{IIs}}{\partial \Delta} = \frac{1}{3(c-2\gamma)} > 0; \quad \frac{\partial N_{OS}^{IIs}}{\partial \gamma} = -\frac{c-2\Delta}{3(c-2\gamma)^2} < 0
\]

• Impact of \( c, \Delta \) and \( \gamma \) on profit

\[
\frac{\partial \Pi^{CCs}}{\partial c} = \frac{(c^2 - \Delta^2)}{9c^2} > 0; \quad \frac{\partial \Pi^{CCs}}{\partial \Delta} = -\frac{2(c-\Delta)}{9c} < 0
\]

\[
\frac{\partial \Pi^{CI}s}{\partial c} = \frac{(c-\Delta)(c + \Delta - 2\gamma)}{9(c-\gamma)^2} > 0; \quad \frac{\partial \Pi^{CI}s}{\partial \Delta} = -\frac{2(c-\Delta)}{9(c-\gamma)} < 0; \quad \frac{\partial \Pi^{CI}s}{\partial \gamma} = \frac{(c-\Delta)^2}{9(c-\gamma)^2} > 0
\]
\[
\frac{\partial \Pi^{IC*}}{\partial c} = \frac{(c^2 - \Delta^2 - 2c\gamma + \gamma^2)}{9(c - \gamma)^2} \leq 0
\]

if \( c \in [\gamma - \Delta, \Delta + \gamma] \) \( \leq \epsilon^d \). Finally, \( \frac{\partial \Pi^{IC*}}{\partial c} > 0 \) when \( c > \Delta + \gamma = \epsilon^d \)

\[
\frac{\partial \Pi^{IC*}}{\partial \Delta} = \frac{-2(c - \Delta - \gamma)}{9(c - \gamma)} < 0 \quad \forall \ c > \Delta + \gamma = \epsilon^d
\]

\[
\frac{\partial \Pi^{IC*}}{\partial \gamma} = \frac{(-c^2 + 2c\gamma + \Delta^2 - \gamma^2)}{9(c - \gamma)^2} \geq 0 \text{ if } c \in [\gamma - \Delta, \Delta + \gamma] \leq \epsilon^d.
\]

Finally, \( \frac{\partial \Pi^{IC*}}{\partial \gamma} < 0 \) when \( c > \Delta + \gamma = \epsilon^d \)

\[
\frac{\partial \Pi^{II*}}{\partial c} = \frac{(c^2 - 4c\gamma - \Delta^2 + 2\Delta \gamma + 3\gamma^2)}{9(c - 2\gamma)^2} \leq 0
\]

if \( c \in [\Delta + \gamma, 3\gamma - \Delta] \leq \epsilon^d \). Finally, \( \frac{\partial \Pi^{II*}}{\partial c} > 0 \) when \( c > \Delta + \gamma = \epsilon^d \)

\[
\frac{\partial \Pi^{II*}}{\partial \Delta} = \frac{-2(c - \Delta - \gamma)}{9(c - 2\gamma)} < 0 \quad \forall \ c > \Delta + \gamma = \epsilon^d
\]

\[
\frac{\partial \Pi^{II*}}{\partial \gamma} = \frac{-2(\Delta - \gamma)(c - \Delta - \gamma)}{9(c - 2\gamma)^2} < 0 \quad \forall \ c > \Delta + \gamma = \epsilon^d
\]

A.6. Proof of proposition 4

Assume in the following that \( c > \Delta + \gamma \) to enable comparisons of welfare levels across all four compatibility scenarios.

Given the level of welfare under the four regimes in Table 2, we can derive the following results:

\[
\Delta W_U|_{(CC,CI)} \equiv W^{CC*}_U - W^{IC*}_U = \frac{1}{6c} \frac{\gamma (c^2 - \gamma c - \Delta^2)}{c - \gamma}
\]  

(C.1)

Since \( c > \gamma \), the sign of this expression is given by the sign of the numerator which is negative for \( c \in [c^1_{(CC,IC)}, c^2_{(CC,IC)}] \), where \( c^1_{(CC,IC)} = \frac{1}{2} \gamma - \frac{1}{2} \sqrt{4\Delta^2 + \gamma^2} \) and

\[
c^2_{(CC,IC)} = \frac{1}{2} \gamma + \frac{1}{2} \sqrt{4\Delta^2 + 2\gamma^2}, \text{ with } c^1_{(CC,IC)} < c^2_{(CC,IC)}.
\]

However, the values of \( c \) that satisfy the duopoly condition are higher than \( c^1_{(CC,IC)} \) and \( c^2_{(CC,IC)} \). This can be demonstrated by showing that \( c^1_{(CC,IC)} \) and \( c^2_{(CC,IC)} \) don’t violate the duopoly condition \( c^d = (\Delta + \gamma) \). Indeed,

\[
\frac{1}{2} \gamma - \frac{1}{2} \sqrt{4\Delta^2 + 2\gamma^2} - (\Delta + \gamma) = \frac{-2\Delta - \gamma \sqrt{4\Delta^2 + \gamma^2}}{-2} < 0,
\]

and

\[
\frac{1}{2} \gamma + \frac{1}{2} \sqrt{4\Delta^2 + 2\gamma^2} - (\Delta + \gamma) = \frac{\sqrt{4\Delta^2 + 2\gamma^2} - 2\Delta}{2} \iff \frac{4\Delta^2 + 2\gamma^2 - (\gamma + 2\Delta)^2}{2} = -2\Delta \gamma < 0.
\]

Finally, \( \Delta W_U|_{(CC,IC)} > 0 \forall \ c > c^d \).

\[
\Delta W_U|_{(IC,CI)} \equiv W^{IC*}_U - W^{II*}_U = \frac{1}{6c - \gamma} (c + 2\Delta - 2\gamma) > 0 \forall \Delta > \gamma \text{ and } c > \gamma
\]  

(C.2)
\[ \Delta W_U|_{\text{CI,II}} \equiv W_U^{CI*} - W_U^{II*} = \frac{1}{6} \gamma \left( c^2 - 3c\gamma - \Delta^2 + 2\Delta\gamma + \gamma^2 \right) \]  
(C.3)

We can easily check that \[ \Delta W_U|_{\text{CI,II}} \left( c_1^{\text{CI,II}} \right) = \Delta W_U|_{\text{CI,II}} \left( c_2^{\text{CI,II}} \right) = 0, \]
where
\[ c_1^{\text{CI,II}} = \frac{3\gamma - \sqrt{4\Delta^2 + 5\gamma^2 - 8\Delta\gamma}}{2}, \]
\[ c_2^{\text{CI,II}} = \frac{3\gamma + \sqrt{4\Delta^2 + 5\gamma^2 - 8\Delta\gamma}}{2}, \]
with \( c_1^{\text{CI,II}} < c_2^{\text{CI,II}} \).

Note that \( \Delta W_U|_{\text{CI,II}} \left( c \right) \to \infty \) when \( c \) tends to \( \infty \). Then \( \Delta W_U|_{\text{CI,II}} \left( c \right) \leq 0 \) when \( c \in \left[ c_1^{\text{CI,II}}; c_2^{\text{CI,II}} \right] \). However, \( c_1^{\text{CI,II}} \) and \( c_2^{\text{CI,II}} \) do not satisfy the duopoly condition.

Indeed,
\[ \left( \frac{3\gamma - \sqrt{4\Delta^2 + 5\gamma^2 - 8\Delta\gamma}}{2} \right) - (\Delta + \gamma) = \frac{\gamma - 2\Delta - \sqrt{4\Delta^2 - 8\gamma + 5\gamma^2}}{2} < 0, \]
\[ \left( \frac{3\gamma + \sqrt{4\Delta^2 + 5\gamma^2 - 8\Delta\gamma}}{2} \right) - (\Delta + \gamma) = \frac{\gamma - 2\Delta + \sqrt{4\Delta^2 - 8\gamma + 5\gamma^2}}{2} \quad \Rightarrow \quad (4\Delta^2 - 8\Delta\gamma + 5\gamma^2) - (\gamma + 2\Delta)^2 = -4\gamma (\Delta - \gamma) < 0. \]

Finally, \( \Delta W_U|_{\text{CI,II}} > 0 \) for \( c > c^d \).

According to these results, we can easily check that
\[ W_U^{\text{CC*}}|_{s=s_{\text{CC}}} > W_U^{\text{IC*}}|_{s=s_{\text{IC}}} > W_U^{\text{CI*}}|_{s=s_{\text{CI}}} > W_U^{\text{II*}}|_{s=s_{\text{II}}} \quad \forall \ c > \Delta + \gamma = c^d \text{ and } \Delta > \gamma \]

A.7. Equilibrium results without public subsidy under the four compatibility regimes

The equilibrium solutions for the proprietary firm and the OS community in the absence of subsidies are readily obtained from \( N_{PS}^k (s), N_{OS}^k (s), p^k (s) \) and \( \Pi^k (s) \) by setting \( s = 0 \), summarized in Table 5\(^{11}\).

<table>
<thead>
<tr>
<th>( p^* )</th>
<th>CC</th>
<th>CI</th>
<th>IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{c - \Delta}{2} )</td>
<td>( \frac{c - \Delta}{2} )</td>
<td>( \frac{c - \Delta}{2} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{c - \Delta}{2} )</td>
<td>( \frac{c - \Delta}{2} )</td>
<td>( \frac{c - \Delta}{2} )</td>
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<td>( \frac{c - \Delta}{2} )</td>
<td>( \frac{c - \Delta}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

| \( \frac{2c\Delta - 7\Delta^2}{4(c-2\gamma)} \) | \( \frac{(c-\Delta)^2}{4c} \) | \( \frac{(c-\Delta)^2}{4(c-\gamma)} \) | \( \frac{2c\Delta - 7\Delta^2}{4(c-\gamma)} \) |
| \( +10c\gamma - 7\gamma^2 \) | \( +10c\gamma + 7\gamma^2 \) | \( +10c\gamma - 7\gamma^2 \) |
| \( -2\Delta\gamma - 3c^2 \) | \( -2\Delta\gamma + 3c^2 \) | \( -2\Delta\gamma - 3c^2 \) |
| \( -4c(V_{OS} + V_{PS}) \) | \( +\Delta^2 - 3c^2 \) | \( -4c(V_{OS} + V_{PS}) \) |
| \( -12cV_{OS} - 4cV_{PS} \) | \( 4c(V_{OS} + V_{PS}) \) | \( -8cV_{OS} - 3c \) |
| \( \frac{2c\Delta - 7\Delta^2}{8(c-2\gamma)} \) | \( \frac{2c\Delta + 8c\gamma}{8(c-\gamma)} \) | \( \frac{2c\Delta - 7\Delta^2}{8(c-\gamma)} \) |

Table 5: n.c. necessary condition for subgame-perfect equilibrium, differences between OSS et PS quality \( \Delta = V_{OS} - V_{PS} \),

Full compatibility (CC), Full incompatibility (II), OSS-compatibility (CI) and PS-compatibility (IC)

\(^{11}\)The calculation of the welfare is omitted for brevity. All calculs are available on request

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A.8. Proof of proposition 5

From Table 4 (Row 7) and Table 5 (Row 6), we derive the following results:

\[
\Delta W^{CC}_U \equiv W^{CC}_U|_{s=s^*} - W^{CC}_U|_{s=0} = \frac{(c - \Delta)^2}{24c}
\]  
(D.1)

\[
\Delta W^{II}_U \equiv W^{II}_U|_{s=s^*} - W^{II}_U|_{s=0} = \frac{c^2 - 2c\Delta - 2c\gamma + 25\Delta^2 + 2\Delta\gamma + \gamma^2}{24(c-2\gamma)}
\]  
(D.2)

\[
\Delta W^{IC}_U \equiv W^{IC}_U|_{s=s^*} - W^{IC}_U|_{s=0} = \frac{c^2 - 2c\Delta + 25\Delta^2}{24(c-\gamma)}
\]  
(D.3)

\[
\Delta W^{CI}_U \equiv W^{CI}_U|_{s=s^*} - W^{CI}_U|_{s=0} = \frac{c^2 - 2c\Delta + 25\Delta^2}{24(c-\gamma)}
\]  
(D.4)

We are now in the position to proof the proposition 5.

The derivative of \(\Delta W^{CC}_U\) with respect to \(c\) is clearly positive. The derivative of \(\Delta W^{II}_U\) with respect to \(c\) can be written as

\[
\frac{\partial \Delta W^{II}_U}{\partial c} = \frac{(c^2 - 4c\gamma - 25\Delta^2 + 2\Delta\gamma + 3\gamma^2)}{24(c-2\gamma)^2}
\]  
(E.1)

This expression is negative when \(c \in [c^{1}_{II}, c^{2}_{II}]\) where \(c^{1}_{II} = 2\gamma - \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2}\) and \(c^{2}_{II} = 2\gamma + \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2}\), with \(c^{1}_{II} < c^{2}_{II}\). Remark that only \(c^{2}_{II}\) satisfies the duopoly condition. Indeed,

\[
\left(2\gamma + \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2}\right) - (\Delta + \gamma) = \gamma - \Delta + \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2} \Leftrightarrow 2\Delta (13\Delta - 2\gamma) + 2\gamma^2 > 0,
\]

and

\[
\left(2\gamma - \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2}\right) - (\Delta + \gamma) = \gamma - \Delta - \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2} < 0.
\]

Finally, \(\frac{\partial \Delta W^{II}_U}{\partial c} < 0\) when \(c \in \left[\Delta + \gamma, 2\gamma + \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2}\right]\) and \(\frac{\partial \Delta W^{II}_U}{\partial c} > 0\) when \(c > 2\gamma + \sqrt{25\Delta^2 - 2\Delta\gamma + \gamma^2}\).

The derivative of \(\Delta W^{IC}_U\) with respect to \(c\) can be written as

\[
\frac{\partial \Delta W^{IC}_U}{\partial c} = \frac{(c^2 - 2c\gamma - 25\Delta^2 + \gamma^2)}{24(c-\gamma)^2}
\]  
(E.2)

This expression is negative when \(c \in \left[\gamma - 5\Delta, 5\Delta + \gamma\right] > \Delta + \gamma = c^{d}\). We easily check that only \(c = 5\Delta + \gamma\) satisfies the duopoly condition. Thus, \(\frac{\partial \Delta W^{IC}_U}{\partial c} > 0\) when \(c > 5\Delta + \gamma\) and \(\frac{\partial \Delta W^{IC}_U}{\partial c} < 0\) for \(c \in \left[\Delta + \gamma, 5\Delta + \gamma\right]\).

Finally, the derivative of \(\Delta W^{CI}_U\) with respect to \(c\) can be written as

\[
\frac{\partial \Delta W^{CI}_U}{\partial c} = \frac{(c^2 - 2c\gamma - 25\Delta^2 + 2\gamma\Delta)}{24(c-\gamma)^2}
\]  
(E.3)
This expression is negative when \( c \in \left[ \gamma + \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2}, \gamma - \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} \right] > \Delta + \gamma = c^d \). We easily check that only \( c = \gamma + \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} \) satisfies the duopoly condition. Indeed,
\[
\left( \gamma + \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} \right) - (\Delta + \gamma) = \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} - \Delta \iff 24\Delta^2 - 2\Delta \gamma + \gamma^2 > 0,
\]
and
\[
\left( \gamma - \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} \right) - (\Delta + \gamma) = -\Delta - \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} < 0.
\]
Finally,
\[
\frac{\partial W_{CI}}{\partial c} > 0 \forall c > \gamma + \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} \quad \text{and} \quad \frac{\partial W_{CI}}{\partial c} < 0 \quad \text{for} \quad c \in \left[ \Delta + \gamma, \gamma + \sqrt{25\Delta^2 - 2\Delta \gamma + \gamma^2} \right].
\]

### Appendix B

#### B.1. Equilibrium values when the government maximizes the total welfare (including firm’s profit)

We present the equilibrium outcome when the government maximizes the total welfare (including the firm surplus). By using the welfare function given in (3), and maximizing for \( s \), we obtain the equilibrium results summarized in Table 6:

<table>
<thead>
<tr>
<th>( c )</th>
<th>( CC )</th>
<th>( II )</th>
<th>( CI )</th>
<th>( IC )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^* )</td>
<td>( c - \Delta )</td>
<td>( c - \gamma - \Delta )</td>
<td>( c - \Delta )</td>
<td>( c - \gamma - \Delta )</td>
</tr>
<tr>
<td>( N_{PS} )</td>
<td>( \frac{c - \Delta}{c} )</td>
<td>( \frac{c - \gamma - \Delta}{c - 2\gamma} )</td>
<td>( \frac{c - \Delta}{c - \gamma} )</td>
<td>( \frac{c - \gamma - \Delta}{c - \gamma} )</td>
</tr>
<tr>
<td>( N_{OS} )</td>
<td>( \frac{\Delta}{c} )</td>
<td>( \frac{\Delta - \gamma}{c - 2\gamma} )</td>
<td>( \frac{(\Delta - \gamma)}{c - \gamma} )</td>
<td>( \frac{\Delta}{c - \gamma} )</td>
</tr>
<tr>
<td>( s^* )</td>
<td>( -c + \Delta )</td>
<td>( -c + \gamma + \Delta )</td>
<td>( -c + \Delta )</td>
<td>( -c + \gamma + \Delta )</td>
</tr>
<tr>
<td>( \Pi^* )</td>
<td>( \frac{(c - \Delta)^2}{c} )</td>
<td>( \frac{(c - \gamma - \Delta)^2}{c - 2\gamma} )</td>
<td>( \frac{(c - \Delta)^2}{c - \gamma} )</td>
<td>( \frac{(c - \gamma - \Delta)^2}{c - \gamma} )</td>
</tr>
<tr>
<td>( W^* )</td>
<td>( \frac{\Delta^2 + 2cV_{PS} + 2\gamma}{2c} )</td>
<td>( \frac{\gamma^2 + 2cV_{PS} - \gamma^2 - 2\gamma \Delta}{2(c - 2\gamma)} )</td>
<td>( \frac{2c + \Delta^2 + 2cV_{PS} - \gamma^2 - 2\gamma V_{OS}}{2c - 2\gamma} )</td>
<td>( \frac{2cV_{PS} + c\gamma + \Delta^2 - \gamma^2 - 2\gamma V_{PS}}{2(c - \gamma)} )</td>
</tr>
<tr>
<td>( n.c. )</td>
<td>( c &gt; \Delta )</td>
<td>( c &gt; \Delta + \gamma ) and ( \Delta &gt; \gamma )</td>
<td>( c &gt; \Delta )</td>
<td>( c &gt; \Delta + \gamma )</td>
</tr>
</tbody>
</table>

Table 6: n.c.: necessary condition for subgame-perfect equilibrium, difference in qualities \( \Delta = V_{OS} - V_{PS} \), Full compatibility (CC), Full incompatibility (II), OSS-compatibility (CI) and PS-compatibility (IC)

**Remark 1** From Table 6, we conclude that a positive public subsidy to OSS users generates a negative total welfare surplus regardless of the compatibility regime. It implies that public subsidies to OSS users are not desirable when the government maximized the total welfare (including the firm’s profit).