Coexisting Multiple Networks mediating Multi-layered Coalition Structure

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Abstract
In this paper, we focus upon a possibility that coalitions with overlapping or multi-layered structure forms, by means of examples from coalition formation game. In particular, utilizing framework of network structure adopted from Page, Wooders, and Kamat (1995), we examine possible effects of various patterns of cost of forming overlapping networks. In some examples, costs deter formation of smaller size coalition in the lower layer rather than large coalitions.

1. Introduction

Coalition is one of the key concepts in cooperative game theory, representing a group of players which has a potential of coordinating their actions inclusive of redistribution of payoffs. Real world counterparts raised in the literature range from a firm, union of sovereign nations, married couple, and voters who voted for the same candidate. Such diverse objectives exhibit difference as well as similarity, and so beyond a certain point, a differentiated treatment would be called forth. Here, we employ network structure inside a coalition as device to get some idea on the internal structure of coalitions and try to show its use via example of overlapping coalition formation with the effect of cost of coalition formation on it.

Myerson(1977) is seminal in proposing linking network structure as a device to represent internal structures of coalitions. There, only players linked directly or indirectly through binary links can act as coalition, or has a potential to do something more that they can when they act individually. This approach captures the interesting aspect that when coalition is formed, its sub-coalition may or may not be formed easily. (See Fig. 1(a).)

However, there seems to be a limitation as well. For instance, coalition tries to act according to agreement, but to carry out the specified action, it is necessary to have mutual monitoring of actions and sanction prepared for the member's non-compliance to the agreement. Then the player who is good at monitoring one player's action may not be the player who has the effective punitive action against that player. This would suggest the structure given by linking network is too simplistic to represent the situation just described. In fact, Myerson (1980) considers an extension of earlier idea by conference structure, where a conference could be thought of as multilateral links within a group of players. Related idea would be that coalition could be identified with a fully linked network. But again under this structure, the concept of "formed" coalition cannot be easily identified, which was one shortfall of the linking network interpretation. (Cf. Fig. 1(b).)

Development of network theory in games and theoretical economics following seminal work by Jackson and Wolinsky (1996), several extensions are proposed. One notable development from our point of view would be the introduction of the intensity of link by Bloch and Duuta(2009). But the generalization proposed by Page, Wooders, and Kamat (2005) seems most appropriate, in that their framework is quite comprehensive and may be extended to distinguish different roles played by the same link at different dates. (also see Kovalenkov and Wooders(2005)) They proposed that the network (with orientation) consisting of multiple types of links (called arcs) and hubs not necessarily corresponding to players, and allowing a loop.

Here, we borrow their framework to add each coalition as a potential hub, and a coalition is said to be formed if all members have a link with the hub corresponding to that coalition. This structure is as if demanding each coalition to establish own headquarter (which may correspond to one particular interpretation of a coalition) and is an excessive simplification from what we described above but may represent one extreme pattern allowing easier derivation of some basic ideas. (Cf. Fig. 1(c).) Especially, in this paper, our analysis is focused upon the effect of the cost of network formation upon the pattern of coalition formation, and this formulation makes such analysis relatively easier. Since early years, the cost of forming coalition was recognized but its analysis remains at the indirect or extremely simplified level. The investigation of network formation supporting coalition would facilitate more detailed and richer analysis and some recent studies touch upon this issue (e.g. Jiang, Theodorakopuolos, and Baras (2006), Kovalenkov and Wooders (1995) and Schjodt and Sloth (1994). Also see Zolezzi and Rudnick (2002).). We follow this literature with multiple layered coalitions.

2. Adaptation from PWK model of networks

Let $N = \{1, 2, ..., n\}$ be the set of players. A coalition S in N ($S \subset N$) is any subset. Let us denote by $A = 2^N$ the set of all coalitions, while for what matters, $A' = 2^N - \{\Phi\} =$"2^N minus empty coalition (denoted Φ)" (equal to the set of nonempty coalitions) may be more relevant, and its cardinality is $2^n - 1$. Denote by B the union of N and A': $B = N \cup A'$.

The set of network G is a subset of $A \times B \times B$ such that A is the set of types of network arc and B is the set of hubs (PWK had no specific meaning imposed on A or B). In particular, we identify (a, i, j) in G with (a, j, i) (i.e. orientation does not matter) and for any (a, i, j) in G, i is not equal to j, so that we do not allow a loop. In this context, the networks considered by Jackson and Wolinsky(1996) are elements in $\{\Phi\} \times N \times N$ satisfying the above restriction and we refer to them as linking networks following PWK. We consider this linking network as a natural candidate of the starting point of network formation in our case. We further restrict G so that if (S, i, j) is a member of G and S is in A'', then i or j belongs to S (for S in $A: S \in A$) and j or i (respectively) is equal to S. This represents that the type S network links players in S to hub S.

What we presume under a network is that if G contains $\{(S, i, S):$ i is in $S\}$ then coalition S is formed, and they can coordinate their action. Along with PWK, Page and

Wooders (2007) advanced a study of much looser coalition which they call a "club". In Page and Wooders, they consider multiple memberships of clubs. Also their analysis is focused on network formation game (with precursor by Bala and Goyal (2000). Also see Demange (2005) and Konishi, Le Breton and Weber (1998)). Also, as for the payoff structure, we utilize cost function which we specify later together with payoffs generated by the underlying strategic game.

3. Multiple coalitions

Aumann and Myerson(1988) analyzed coalition formation via network formation based on the Myerson(1977)'s analysis of network structure where payoffs are specified by a characteristic function and ultimate incentive is prescribed by a cooperative solution concept.

Recently, coalition formation received a renewed interest where ultimate incentives are given by partition function or in some occasions, semi-non-cooperative solution concept called coalitional equilibrium (see Bloch (1996) and Ray and Vohra (1999)). We combine coalition formation game with coalition formation game, and show that there is a possibility that coalition might be formed within a coalition. This framework further suggests a possibility that a formation of coalition which intersects with two mutually disjoint coalitions.

As an example, we consider a bargaining model to split a dollar among n players. Bargaining protocol is given by a sequential bargaining game a la Binmore-Stahl-Rubinstein. But before bargaining starts, players can form a coalition so that they coordinate their actions in the course of the sequential bargaining game. However, we consider a situation where the redistribution of money among the coalition members can take place only after the bargaining stage (known as the assumption of incomplete contract). I.e. we assume that they cannot commit to a way of dividing coalitional earning among them ex ante, in a sense that a renegotiation in a later stage cannot be avoided so that any agreement made in advance could be overturned. (One may argue that this assumption itself could be derived from some limitation in the property of network, but we do not have an answer right now). Thus they are eventually going to hold their own bargaining among themselves again. And we assume that prior to bargaining, they can form a (sub-)coalition also. Thus we have potentially the situation of multiple coalitions (along the time axis and also as the following example indicates, sub-coalitions are distinguished from the coalitions they belong to). Further, it is natural that as far as more than or equal to three players remaining in a coalition, this possibility to form internal coalition remains, and hence we assume that all such possibilities are exploited. Games are played in stages t=0, 1, …, and we consider coalition formation stage t for game played in stage t+1 (or T(t) in general), we obtain Q(t), the coalitions formed at stage t, given by a partition of N. In this example, for t > t', elements of Q(t) gives rise to a partition of some element in Q(t'), but this is not necessarily the case as the public good example shown below would indicates. Below, we write G(t) for the collection of coalitions formed prior to t. (here we do not consider the possible dissolution of coalition, which is another restriction.)

Analyzing this process, one can show that in the limit coalition structure (as the time interval between any successive moves of the game vanishes) of stationary subgame perfect equilibrium outcomes, any formed coalition of size more than two would be subdivided into sub-coalitions so that in terms of the finest partitions induced by sub-coalitions, the size of the maximal coalition is two. In the next section, we add the assumption that for any coalition to function, network structure to support it is necessary.

Below we shall briefly sketch one leading example. We try to minimize technical description of the analysis.

The basic game we consider is a sequential bargaining game to split-a-dollar which we write $g\left(1;h(1)\right)$ where h indicates parameters determined by the history before this game takes place. This game starts with player 1 proposing a division of a dollar among n players, x, then players 2, 3, in that order replies by "Yes" or "No". If all players say "Yes" then the division x realizes immediately. If some player replies by "No", then player 2 makes next proposal, with players 3,4,…,n,1 replies. The rest of the rule is the same as above. If the game continues without stopping, then impasse results which is equivalent to receiving 0 for each player. Players evaluate the outcome x by the utility function $u_i(x_i) = x_i^{a_i}$ where $0 < a_i < 1$. We only look at the limit stationary subgame perfect equilibrium outcomes.

Before the sequential bargaining game, players have an opportunity to form a coalition. This stage is played as a coalition formation game which we write $g\left(0; g(1); h(0)\right)$ (with some omission of variables). In this game, player 1 first proposes coalition S (including 1), and members of S replies by "Yes" or "No". If all members say "Yes", then "S forms" and if $N = S$, then the stage ends to move to $g(1)$. Otherwise, the player with the least number in N-S makes next proposal S' in $N - S$. The rest is the same as above. (One may have to invoke equivalence up to modulo n; i.e. $k = m (\bmod n)$ if $k - m = zn$ for some integer z.) As declared above, denote by $Q(0)$ the partition of N resulting from the limit stationary subgame perfect equilibrium coalition structure of this stage. If $Q(0)$ involves only singletons, then after $g(0)$, nothing happens. If there is S in $Q(0)$ with $\left|S\right| > 1$, then after $g(1)$, there shall be a bargaining inside S to split the coalitional proceed earned in the basic bargaining game, i.e. $x(S) = \sum_{i \in S} x_i$ (the sum of money earned by the members). We call this a coalitional bargaining stage and refer to it as $g\left(3; Q(0), S, x(S), h''(3)\right)$. The rule of the game is the same as the basic bargaining game.

Further, we assume that players can form coalition prior to each coalitional bargaining stage. For instance, players in S can form a coalition prior to $g\left(3; Q(0), S, x(S), h''(3)\right)$ and after this game within formed coalitions, they bargain to redistribute the proceeds from the coalitional bargaining stage, too. (We save t=2 for this stage.) And stages can keep increasing as far as there is a room to form coalition, like they bargain at t = 5 but they could form coalition for that bargaining stage at t = 4, etc.. In fact, we can show that if the sub-coalition structure consists of singletons only, then there shall be no further stage. Also one can impose that if the coalition formation results in the same coalition remaining, then there shall be no further coalition formation stage, so that the number of the stages is guaranteed to be finite.

Next, we describe the limit stationary subgame perfect equilibrium allocation in terms of monetary distribution.

When m players with coalition structure $Q=\{S_1,S_2,...,S_k\}$ with $|S_i|=s_i$, $a_{S_l}=\max\{a_i; i\in S_l\}$, and the amount of money to be split among them is x dollar, then the limit outcome of the game is given by

$$x_i=\frac{s_j a_{S_j}}{\sum_{S_{j'}\in Q} s_{j'} a_{S_{j'}}}\frac{a_i}{\sum_{i'\in S_j} a_{i'}}x$$

for any i in S_i in Q. Here, due to the rule, delegation of bargaining power to the toughest player takes place which drives coalition formation in this case.

Based on this formula, one can show that provided that utility functions are not identical, and whenever there are more than two players, (nontrivial) coalition forms.

As a specific example, consider the case with $n=4$, $a_1=1$, and $a_2=a_3=a_4=a<1$. Then at the first coalition formation stage, they have options to form a 2-person coalition or 3-person coalition.

If 2-person coalition, say, {1, 2} forms, then the allocation is given by

$$x_1=\frac{1}{(1+a)^2}, x_2=\frac{a}{(1+a)^2}, x_3=x_4=\frac{a}{2(1+a)}$$

If 3-person coalition, say {1, 2, 3} forms, then the coalitional payoff of the coalition {1, 2, 3} is given by $x(\{1,2,3\})=\frac{3}{3+a}$ and sub-coalition, say {1, 2} forms, then the final outcome would be

$$x_1=\frac{6}{(3+a)(2+a)(1+a)}, x_2=\frac{6a}{(3+a)(2+a)(1+a)}, x_3=\frac{3a}{(3+a)(2+a)}, x_4=\frac{a}{(3+a)}.$$

From this, we conclude that players 1, 2, and 3 prefer the latter allocation, and hence once three person coalition {1, 2, 3} forms and then a sub-coalition {1, 2} forms.

4. Costs

Costs of adding new network reflect several possibilities. In general terms, creating a coalition S within T means establishing new network with a hub "S" and cost for it could depend upon many things, especially which networks exist and for what purpose, i.e. in the current context, for what "stage" game. Thus, the cost for creating new network with a hub "S" for player i in S is given by a function $c(i,S,T;G(t))$

where as noted earlier, $G(t)$ is the networks existing at stage t. Given the assumption that a singleton coalition requires a creation of its own network, and that for each coalition formation stage, there emerges a sub-coalition structure, so that $Q(0)$ is the coalition structure at stage 0 which is a partition of N, while for $Q(t)$ with $t = 2m > 0$, $Q(t)$ is a partition of N and refinement of $Q(2m-2)$.

If there is a scale economy at coalitional level, then the costs for establishing networks serving the same members shall be smaller. Similarly, if there is an economy associated with establishing network for a subset of players for whom already there is an established network, then the costs to establish these networks are cheaper than other occasions. In this context, scale economy working at link level has some difficulty as they must go through new hub but one may impose that the costs of establishing new links through new hub connecting the same pair to be smaller. In the extreme, one may assume these additional costs to be 0. Also for a singleton coalition, one may assume that the cost of establishing new hub is 0 or extremely cheap.

On the contrary, one may claim that there is a scale diseconomy. This may be the case if handling several agreements is more costly for a member. Especially when a subset of player for whom already there is a network may be costly, as distinguishing members of new network from the rest may be costly under some circumstances. We shall discuss this case later.

One simplest assumption may be constant returns so that costs of establishing a network only depends upon the number of players involved. As seen from these various properties this function may possess, this formulation could accommodate several different suppositions arising from observations of real world.

One quick review of the effect of costly network formation on the equilibrium of coalition formation game is given along the example given above. Suppose the constant return to scale cost function. As the cost rises from 0, the cost to form two networks for a member of S in the above example becomes costly. Then at some cost level, players' incentives to form a network twice are hampered by the costs of forming networks, and hence there emerges an incentive to choose networks embodying a lower degree of hierarchy at the inception of the game. And there may be the cases that players choose to settle for different network structure from the pattern yielded if there is no cost in forming network at all. (This may depend upon the timing of costs incurred too.) If there is a return to scale in costs of forming new networks at the sub-coalition level, then this phenomenon would be restrained.

For our leading example, let us assume that forming any coalition cost network cost $c > 0$ in monetary units per player. Further, suppose that these costs are subtracted from individual payoffs before the bargaining reaches (and hence becomes sunk). Then when {1, 2} forms, the allocation is given by

$$x_1 = \frac{1}{(1+a)^2} - c, x_2 = \frac{a}{(1+a)^2} 1 - c, x_3 = x_4 = \frac{a}{2(1+a)} - c$$

If {1, 2, 3} forms and when sub-coalition {1, 2} forms, the final outcome becomes

$$x_1 = \frac{6}{(3+a)(2+a)(1+a)} - 2c, x_2 = \frac{6a}{(3+a)(2+a)(1+a)} - 2c, x_3 = \frac{3a}{(3+a)(2+a)} - 2c, x_4 = \frac{a}{(3+a)} - c$$

Also we assumed away that for the final bargaining stage within each coalition, where all the sub-coalitions are singletons, no cost is incurred.

Above formula indicates that if

$$c > \frac{a^2(1-a)}{2(1+a)(2+a)(3+a)}$$

holds, then player 3 prefers not to join the 3-person coalition, and hence only 2-person coalition forms, instead.

5. Linking network with costly shutdown

As an alternative, one may represent our idea of multiple coalitions through a linking network (with players as only hubs) by allowing hubs to shut down certain links at a time. For instance, one may employ Myerson type coalitions on network. When called for, if members of certain coalition are connected through the existing network, then they can form this coalition and coordinate their actions through the relevant part of the network, while the link with rest of network remains shut down by members intentionally shut down the connection. Note that without the notion of shut-down, one cannot distinguish between formed and active coalitions and potential coalitions in our context.

One interesting and also complicated aspect of Myerson(1977)'s formulation is the way in which hubs connected affect the ease of shutting down and forming new coalition because there are many patterns of connection inside a coalition. Therefore, players must be very careful in establishing new link, because it affects future course of coalition formation more explicitly than in other formulations. This also makes analysis bit involved. By contrast, fully connected network representation of coalition allows relatively easier way for our analysis. In fact, to form k person sub-coalition of an existing n-person coalition implies that each of k members must shut $n-k$ links and hence, in total, $k(n-k)$ shutdowns must be carried out. Similarly, if m person coalition to be formed while k person among m players in the existing n person coalition implies costs incurred are shut-down of $k(n-k)$ links with creation of $m(m-k)$ links.

One interesting aspect of shut-down interpretation is a sort of coalition formation problem starting from fully connected networks of the entire group. Note that normally coalition formation, and to that effect, network formation is analyzed often from the null connection situation. But for the situation corresponding to bargaining problem and also coalition formation problem, insofar as communication network is concerned, it is supposed that all players are linked, in the sense they participate in the same game. In this sense, there are connections among all of them. Therefore, starting out with fully connected network and thinking of shutting down existing network for the sake of coalition formation may be of some interests. Given n players fully connected, creation of m person sub-coalition would imply shutting down of m(n-m) links. From this, one can easily observe that the cost peaks at n/2. Thus as far as social cost is concerned, creating mid-sized sub-coalition is the most costly, whereas individual cost is merely decreasing with the size. In this sense, a larger coalition is relatively cheaper to form, and depending on the benefit, there could be a tendency that socially too large coalition to be formed.

Up to now, we are not specific about reason why players do not wish to use the existing network when they act as coalition. As we noted earlier, this is not always the case, but there would be occasions where this may be true. As some of those occasions, we list the case where there is a specific information to be conveyed through the network on the one hand. On the other hand, there is a possibility that this information conveyance is deterred or this information is leaked through the network when linked to non-members. In the former case, players outside the coalition could disturb the information transmission by letting the network congested intentionally. In the latter case, they may tap the information (assuming the leaky network) and use them to their advantage (and so to the coalition's disadvantage). In the context of non-cooperative game theory, one may ask if there would be any information leakage which would cause damage in general, because in equilibrium, all the relevant information to be conveyed concerning strategy must be predictable, and so only in some imperfect information game or when mixed or correlated strategies are called for, information tapping becomes problematic. (As another source for players desire to have separate network would be the case where link itself is payoff relevant, like the case of marriage game where link implies marriage.)

5. Public goods

As another example, consider the game of voluntary provision of public goods (based on Ray and Vohra (2001)). Again there are n players with Player set N. Each player chooses the level of privately provided public good, x_i. Payoff of player i given (x_j) is

$x - \frac{(x_i)^2}{2}$, where $x = \sum_j x_j$.

Players choose x_i simultaneously, and prior to this stage, players can form coalition as before. Given a coalition S formed, members of S are assumed to maximize the sum of payoffs of the players in that coalition (without redistribution). In the coalitional equilibrium given a coalition structure Q, each player in the coalition S with size s chooses x_i = s.

It is well known that for $n = 4$, player 1 forming a singleton and the rest of players forming a 3-person coalition is the stationary subgame perfect equilibrium outcome. (To verify this, compare the payoff with a grand coalition N, which yields 8 for each player, while coalition structure with {1} and {2, 3, 4}, where 1 provides 1 unit and the 3-person coalition provides 9 yielding payoff 9.5 to player 1 and 2 each to the rest.)

One example of cost of shutdown can be illustrated here. Suppose forming size m coalition costs (n-m)c per player. Then for player 1 to commit to singleton, it must shut down 3 connections, costing him/her 3c. (After this 3 players may free-ride in saving cost of shutdown, and so 3-person coalition should emerge for sure.) If c is greater than 0.5, then apparently player 1 prefer grand coalition which can be formed at no cost. For the simultaneous version of the game, it is known that there are two Nash equilibria with 2-person or 3-person coalition forms when there is no cost involved. The equilibrium with 2-person coalition is supported by the balance that one outsider is indifferent between joining the 3-person coalition and remaining an outsider to the 2-person coalition. As soon as shutdown cost is considered, immediately the equilibrium with 2-person coalition disappears (as opposed to the case of cost increasing with the size).

Now, getting back to the story, and if we assume that coalitional redistribution is possible on time, and we add the coalition formation stage prior to this coalition formation game with the same rule, then one can show that players form the grand coalition N so that player 1 does not form a singleton in the second coalition formation stage. (For instance, in the above example, if a grand coalition forms prior to the coalition formation stage for the public goods provision game stage, then it can offer player 1 a payment of 1.8 by the rest of players so that player 1 proposes the grand coalition rather than the singleton in the next stage. The rest of players have an incentive to do so, because paying 0.5 each would bring an increase in payoff by 6. (One big difference from the previous example is that the commitment to redistribution is assumed to be possible in this case. See Imai and Horie(2002).) Thus this is an example that forming a grand coalition prevents subsequent coalition formation.

However, when the second coalition formation stage employs random proposer protocol so that with an equal probability, each player becomes a proposer, then an upfront payment of subsidy to the prospective proposer does not help prevent singleton formation. (This is because it is assumed that the amount of redistribution must be specified and committed to at the time of coalition formation, and hence everybody must receives more that 9.5 which is impossible.) Nevertheless, if they can arrange a contract so that upon knowing the identity of the proposer, the rest of players agree to pay that player the subsidy, then again prevention of the commitment by singleton formation becomes possible. (A related concept of contingent coalitional contract is discussed in Bloch and Gomes (2006)) So it is important to have the communication network as well as timely assurance of the payment. This could be the case of a more complex role played by the network.

Now, let us introduce a cost of network depending upon the role it plays. Let c be the per capita cost of establishing a network for a coalition as before, while for establishing network with a higher degree of functioning, it cost c' > c. Then player's expected payoff when no prior formation of the grand coalition, and hence coalition formation at the second stage, is $\frac{9.5+3\times 2}{4}-c$, while that of forming coalition in the first stage with higher functioning is $8-c'$. Thus depending on the magnitude of c' – c, coalition formation may take different pattern. Also if there is an element of risk aversion, the odds for the grand coalition rises, showing the risk pooling aspect of coalition formation.

6. Concluding remark

There are many networks in our real life, and so it is a fact that there are several networks coexisting. Obviously, many of these networks differ in their functions, although it is notable that some networks may have different patterns of connection, and in particular, their components may not exhibit any mutually inclusive relationship.

In this paper, we focus upon a possibility that coalitions form along the time axis with possible multiple memberships, by means of examples from coalition formation game. In particular, utilizing specific network structure adopted from Page, Wooders, and Kamat (2005), we postulated a specific network structures and examine possible effects of several patterns of cost of forming coalitions. What we have dealt with is a few simple examples, but we believe that they indicate the potential value for further investigation.

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