Efficiency and vertical networks: A note on demand uncertainty and separated markets

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Abstract
We develop a model that captures demand uncertainty and separated markets to identify a trade-off between stability and profitability possibly existing in vertical networks. In particular this paper develops a framework to marriage the idea of (idiosyncratic) demand shocks with the consideration of the competitive environment of the firms in a buyer-seller network. We consider a duopoly model with price competition and a given degree of product differentiation to model separated markets on the buyers' level. Buyers operate without the possibility of direct market interaction. Bundling of the demand for inputs enables the buyers to realise economies of scale from a collective point of view. In case of bargaining on cooperation surplus inefficiencies arise because buyers realizing comparatively large positive demand shocks don't internalise externalities of their strong bargaining power due valuable outside options. In particular they don't consider demand effects of comparatively higher input prices for buyers confronted by a lower density of consumers. In consequence inefficiencies arise from network perspective.

Keywords
Buyer-Seller network, Cooperative organization, Demand uncertainty, Club good.

1. Introduction

Recent years have witnessed increasing interest in the determinants of firms' organizational choices. In particular questions relating to cooperation among firms have been frequently analyzed. New models of economic exchange namely networks have been developed. Vertical networks as supply structures vary across industries and are distinct from vertically integrated firms and markets. By forming

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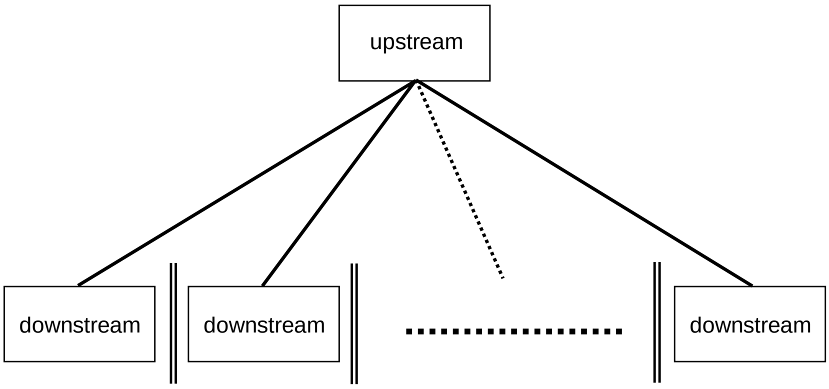
vertical collaboration structures, however, firms alter the competitive position of several firms and in turn influence market structure and performance. This two-way flow of influence is central in our analysis.
Thus the intention of this paper is to identify a trade-off between stability and profitability possibly existing in vertical networks. We develop a model that captures demand uncertainty and separated markets.
The remaining part of the paper is organized as follows. Section 2 provides a brief overview of the related literature in order to motivate our approach. The model is presented in section 3. In section 4 we explore efficiency properties of buyer-seller networks. Section 5 concludes.

2. Related Literature

This paper is a contribution to the literature of group formation and cooperation in oligopolies. Questions of group formation and cooperation have long been in focus of economic research especially in game theory. A central issue related to the more formal theory of group formation is the formulation of a proper coalition game which assigns cooperation rents to a given set of players and to every subset of players. A coalition game also specifies pay-off functions for every player and strategy. Stability and dimension of coalitions depending on different pay-off functions und cooperation rents are important aspects discussed in this context (Myerson 1977; Bloch 1995; Bloch 1997). The described coalition approach only indirectly analyses specific relationships between members of a coalition. The characteristic function assigns to every subset S of the players set N the payoff that can be realized through collaboration of the members of S independently from outside players $N\backslash S$. In this setup symmetric relationships are implicitly assumed: every firm who is part of a coalition cooperates with every firm who is member of the same coalition. If we in turn allow for cooperative relationships that are exclusive, asymmetric structures of cooperation will be generated that are different from those studied in the coalition-formation literature. If there is cooperation between two firms we will call this relationship a “link”. A network can be defined as a set of firms related with a set of pair wise links between the firms (Jackson 2003; van den Nouweland 2003). In this context a star network is a structure of cooperation with a central firm directly linked to every firm while none of the other firms have a direct link with each other. To study concrete problems, industrial structures are often interpreted as networks in the sense above (Economides and Himmelberg 1995; Hendricks et al. 1995; Backerman et al. 1996). For example, recent years have witnessed a large body of literature regarding to buyer-seller networks (Kranton and Minehart 2001; Holmström and Roberts 1998). Questions respecting to advantages of vertical networks in comparison to vertical integration and respecting the influence of different economic scenarios on the formation and optimality of buyer-seller networks have been studied recently. Importance and economic consequences of demand shocks in vertical buyer-seller networks has been an object of analysis too. Kranton and Minehart (2000) show that networks can yield greater social welfare when manufactures experience large idiosyncratic demand shocks. They also highlight comparative advantages of networks: capacity sharing and flexibility. Kranton and Minehart (2000) argue vertically integrated buyer that suffers large negative shocks may regret having built costly unused productive capacity. In networks exists fewer units of productive capacity and buyers suffering the largest negative shock do not procure inputs. Inputs are allocated flexible to the buyers with the highest realisation of valuation for such an input. Therefore incentives for the formation of vertical networks exist. The paper of Kranton and Minehart (2000) is a refinement of Piore and Sabel's (1984) work on „flexible specialists". Piore and Sabel (1984) argue that networks emerge in times of greater economic uncertainty. The connections between demand uncertainty and industry structure have also been in focus of earlier papers (Baron 1971; Holthausen 1976). In addition to different kinds of uncertainty, the competitive environment of the network firms is in focus of some papers (Goyal and Joshi 1999, Leahy and Neary 1997). Goyal and Moraga-González (2001) analyses the connection between competitive environments, incentives to invest in research und development (R&D) und the structure of the network. They show that in absence of rivalry between the network firms in separated markets the complete network, characterised by the existence of links between every member of the network, is stable, profit maximizing und socially optimal. In case of strong rivalry in a model of Cournot competition in a homogenous product market Goyal and Moraga-González (2001) show that the complete network is stable, but intermediate levels of collaboration and asymmetric network structures maximizes industry profits and welfare.

This paper develops a framework to marriage the idea of (idiosyncratic) demand shocks with the consideration of the competitive environment of the firms in a buyer-seller network as a vertical industry structure. The several buyers on the downstream level operate in separated markets without the possibility of direct market interaction. All of them need a uniform specific input factor to produce the final product. This input doesn't have to be a purely private (rival) good. The downstream firms could establish cooperation (network) for bundled procurement of this specific input (Carr and Landa 1983; Cooter and Landa 1984). In particular they could found an upstream firm (production club) for bundled production of the specific input. Therefore closeness of this issue to the theory of the clubs according to Buchanen (1965) und Olson (1965) seems natural. Bundling of the demand for inputs enables the upstream firm to realise economies of scale for the network from a collective point of view. The demand of every downstream firm for the homogenous input determines the cost savings per unit and therefore the cooperation rent in the network. Questions relating to scale effects und cost sharing arise naturally (Kaplan and Wetterstein 1999; Oughton and Whittam 1997; Moldovanu 1996). The figure below shows the described setup.

Figure 1: industrial structure



In this paper we assume zero-profits for the upstream firm. Therefore possible cost savings on the upstream level are completely redistributed to the downstream level. In context of the literature above the upstream firm can be interpreted as a joint project of the downstream firms with the aim to reduce marginal costs of input procurement. Already without demand uncertainty questions regarding proper distributions of the cooperation rent in shape of cost savings arise. Each downstream firm face oligopolistic competition but its market is separated from the other downstream firms of the network. Demand for input which could be produced by the upstream firm depends on the market success of the downstream firms in the network. Idiosyncratic demand shocks influence downstream markets randomly. Therefore the demand for inputs possibly produced by the upstream firm is random too. Is there a set of stable divisions of the cost savings? Can we identify divisions that firms would agree with ex ante? How should cost savings been allocated from a collective point of view? In case of some downstream firms experience positive demand shocks while other downstream firms experience negative demand shocks or no demand shocks asymmetric demand for inputs arises in the network. In consequence asymmetric contributions to the realized economies of scale in shape of cost savings on the upstream level arise too. Should costs savings been allocated asymmetric from network perspective? Should we strengthen differences in the network or should we balance between the asymmetric downstream markets with proper divisions of the cost savings? These questions will be studied in the submitted paper. At first we present the model.

3. Model

3.1 The basic model: industrial structure and costs

Consider a set of $I=\{1,\dots,N\}$ firms. The companies $i\in I$ face price competition in separated downstream markets in presence of a given degree of product differentiation. To differentiate products across from their competitors the downstream corporations $i\in I$ need the specific input factor *G* to produce there respective final good. Firms $i\in I$ cooperate by forming network structures to bundle the demand for the specific input good to realise economies of scale. The input good *G* doesn't have to be physical. One can think about back office activities or IT- Service. The production function of firm $i\in I$ is given by:

$$q_i : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+,\ (x_i, G) \mapsto q_i(x_i, G)$$

where q_i represents the output and x_i is the quantity of an additional input factor. As introduced above *G* denotes the amount or quality of a specific input factor. We further assume for all downstream firms a relation of complementarily of 1:1 between the input good *G* and the output that is sold on the market. Production of *G* causes fix cost to the amount of $F\in\mathbb{R}^+$. We do not assume *G* to be a purely (rival) private good. In the following $\alpha\in[0,1]$ represents the congestion parameter related to the use of *G*. In case of $\alpha(0,1]$ additional costs $\kappa(\alpha)$ occur to maintain the quality *G* for varying q_i. It is supposed that the production of the input good *G* in the network is given by the following equation (Jaramillo et al. 2003; Barham et al. 1997):

$$G = \frac{F+\kappa(\alpha)}{e^{\alpha\cdot\sum_{i=1}^{n} q_i}}, \quad \alpha\in[0,1]; \quad \kappa(\alpha) = F\left(e^{\alpha\cdot\sum_{i=1}^{n} q_i} - 1\right)$$

Note that if $\alpha = 0$ the (club) good *G* is in fact a purely public good (If α=1 the (club) good G is a purely private good). Therefore *G* can be used in the own production function by downstream firm *i* without externalities for other firms of the network. In this situation $\kappa(\alpha)=0$ is implied. Consequently production costs per unit of *G* arise to the amount of: $(F/q)\cdot e^{\alpha q}$. This implies minimal costs per unit if $q=1/\alpha$ holds. In this paper we assume $q<1/\alpha$ to be satisfied generally. In particular $q_i<1/\alpha$; $(q_1+\ldots+q_n)<1/\alpha\ \forall q_i$ is supposed. This implies always decreasing average production costs of *G* in case of joint production. Remember that firms operate in separated markets. Therefore under the assumption of profit maximizing firms joining the network *I* satisfies the criteria of both individual and collective rationality for any sub-coalition of downstream firms. Therefore the network $I=\{1,\dots,N\}$ captures all firms who need the specific input good *G* in the society. For that reason a possibly existing (network) extern supplier is not able to produce *G* to a lower amount of costs. Thus to ensure non negative profits for the (network) extern producer the market price p_M has to satisfy $p_M \geq (F/q_i)\cdot e^{\alpha\cdot q_i}\ \forall q_i$. The condition implies that downstream firms are always better off with the own production of the input *G* compared to procurement on (network) extern markets. We assume the only costs occurring are expenses to produce or procure the input good *G*. In case of own input production only costs to the amount of $(F/q_i)\cdot e^{\alpha\cdot q_i}\in\mathbb{R}^+$ per unit arise. In particular costs of the inputs x_i are not considered. Procurement of the input good on (network) extern markets causes costs per unit denoted by $p_M \in \mathbb{R}^+$. Alternatively downstream firm *i* can procure the input from the (network) upstream firm with costs of $p_i^u \in \mathbb{R}^+$ per unit. Procurement of the input *q* from the (network) upstream firm causes costs $F \cdot e^{\alpha \cdot q} \in \mathbb{R}^+$ faced by the upstream firm. In this framework fix costs can be widely interpreted. Examples are economies of scale or innovation technologies. The assumption of a convex innovation technology is in line with the approach of d'Aspremont and Jacquemin (1988) and Tirole (1992). Both papers assume increasing costs for the accumulation of further units of experience. Therefore the accumulated stock of experience is convex in investments in research and development. The upstream firm is able to produce the input good for several downstream firms bundled. Therefore the average production costs of the input good are in shape of:

$$K^U : (0,\infty) \to \mathbb{R}^+;\ Q \mapsto K^U(Q) := \tfrac{F}{Q} \cdot e^{\alpha \cdot Q} \quad \tfrac{\partial K^U}{\partial Q} < 0;\ Q = \sum_{i=1}^{n} q_i^u;\ q_i^u \in \{0, q_i\}$$

In this specification Q represents the sum of demand for the input good from the *N* downstream firms derived from their individual (and separated) market situation. Within this specification it is implicitly assumed either the whole demand for input is passed or zero units are passed to the upstream level by the downstream firms. For the upstream firm the zero-profit condition holds. Therefore all cost savings in shape of scale effects are given back to the downstream level. In the symmetric case $c_i = c_j$ for all downstream firms *i,j* holds. Therefore the shape of marginal costs c_i of downstream firm *i* is characterized as follows:

$$c_i := \begin{cases} F \cdot e^{\alpha \cdot q_i} \cdot \alpha & \textit{own production of the input} \\ p_M & \textit{procurement on (network) extern markets} \\ p_i^u & \textit{buying from the (network) upstream firm} \end{cases}$$

To derive demand, prices and profits in equilibrium we have to model the downstream markets.

3.2 Downstream profits without demand shocks

We consider a duopoly model with price competition and a given degree of product differentiation to model the *N*= {1,…,*n*} markets on the downstream level. In order to allow for price competition with the possibility of heterogeneity among firms and profits, a model of spatial competition called "linear city" is used (Launhardt 1885; Hotelling 1929; d'Aspremont et al. 1979). The city consists of a street of length 1. There exist two firms and they sell their output on a single market and compete in prices (Bertrand-competition). We assume separated downstream markets. Therefore every firm on the downstream level face this setup and the competitor of every firm is not member of the network. Consumers live along the street and are uniformly distributed with the density ω. Thus the total number of consumers is equal to ω. Each consumer wants to buy exactly one unit of the good

or nothing at all, if the price exceeds his surplus from consumption $\overline{\upsilon} \in \mathbb{R}^+$. Individuals only differ in taste specified by the spot, where die individual is situated which is labelled by *q*, $0 \le q \le 1$. Consumer *q* buys at firm 1 if the total costs are lower than if buying at firm 2 and total expenses do not exceed his valuation $\overline{\upsilon}$ for the good.
For every price combination p_1 and p_2 we can find the consumer who is just indifferent from which store to buy. The marginal consumer is denoted by $\hat{q} = \hat{q}\,(p_1,p_2)$. He is located at the point where his total costs that include price und transportation costs are equal irrespectively of where he buys the good such that $\delta T(\hat{q}) + p_1 = \delta T(1-\hat{q})\, p_2$ holds, where $T:[0,1] \to \mathbb{R}_0^+,\ \ q \mapsto T(q),\ \ T(0)=0$, $0 < T'$, $0 \le T''$, $\forall q \ne 0$ is the common transportation cost function; $\delta \in [0,1]$ denotes a parameter that captures the degree of differentiation in the market. For $\delta = 1$ the model can be interpreted as second stage of a Hotelling model with endogenous product choice that leads to maximal differentiation (Hotelling (1929); d'Aspremont et al. 1979). p_j, $j \in \{1,2\}$ is the price to be paid for the good chosen from firm 1 and firm 2. The marginal consumer is important to derive the two firm's market demand functions: All consumers located to the left of $\hat{q}$ buy from firm 1 and all consumers located to the right of the marginal consumer buy from firm 2. Assume that the maximum willingness to pay $\overline{\upsilon}$ is high enough that every individual buys in equilibrium. Recall that this setup holds for all of the *N* downstream markets. Therefore all firms at the downstream level face oligopolistic price competition with a given degree of product differentiation.

Remark 3.2 (Shape of demand functions): *If both companies serve some customers, the demand functions are strictly decreasing in the own and strictly increasing in the competitor's price. A priori it is impossible to identify the sign of their second derivatives. It is determined by the second and third derivatives of to transportation cost function. Note that a linear or quadratic transportation cost function implies that the demand functions are linearly decreasing (increasing) in the own (competitor's) price and consequently the second derivatives of the demand functions vanish:* $\frac{\partial^2 q_j}{\partial p_k \partial p_l} = 0;\ j,k,l \in \{1,2\}$

Proof: An increase in the own price leads to a lower market share (and vice versa for a price increase of the competitor):

$$\frac{\partial q_1}{\partial p_1} = \frac{-1}{\delta\left[T'(q_1) + T'(q_2)\right]} = -\frac{\partial q_2}{\partial p_1} < 0, \qquad \frac{\partial q_1}{\partial p_2} = \frac{1}{\delta\left[T'(q_1) + T'(q_2)\right]} = -\frac{\partial q_2}{\partial p_2} > 0.$$

$$\frac{\partial^2 q_1}{\partial p_1^2} = \frac{T''(q_1) - T''(q_2)}{\delta\left[T'(q_1) + T'(q_2)\right]^2} \frac{\partial q_1}{\partial p_1}, \qquad \frac{\partial^2 q_1}{\partial p_1 \partial p_2} = \frac{T''(q_1) - T''(q_2)}{\delta\left[T'(q_1) + T'(q_2)\right]^2} \frac{\partial q_1}{\partial p_2}$$

$\frac{\partial^2 q_j}{\partial p_k \partial p_l} = 0;\ j,k,l \in \{1,2\}$ if T'' is constant (which is the case if *T* is linear or quadratic). The same holds for firm 2.□

In this paper the frequently used examples of linear and quadratic transportation costs are applied to prove the existence of several results.

Example 3.2 (linear and quadratic transportation costs): Assume that the transportation cost function is linear or quadratic $T(q):=\alpha+\beta q+\gamma q^2$ the marginal consumer, demand and profits for the downstream firms is given with $c_i = F \cdot e^{\alpha \cdot q_i} \cdot \alpha \quad \Leftrightarrow \quad \psi = 1$ sonst $\psi = 0$, $i \neq j \in \{1,2\}$ $d \in \{d_q, d_l\}$: *quadratic:* $0 \leq \alpha, \beta,\ 0 < \gamma,\ d_q := \beta + \gamma$; *linear:* $0 \leq \alpha,\ 0 < \beta,\ \ 0 = \gamma,\ d_l := \beta$:

$$\hat{q} = \frac{p_2 - p_1 + d\delta}{2d\delta};\ q_i = \left(\frac{p_j - p_i + d\delta}{2d\delta}\right)\omega;\ p_i = \frac{1}{3}c_j + \frac{2}{3}c_i + b\delta;$$

$$\pi_i = \frac{\left(\frac{1}{3}c_j - \frac{1}{3}c_i + d\delta\right)^2}{2d\delta}\omega - \psi F e^{\alpha \cdot q_i}$$

Proposition 3.2 (Comparative static's): If transportation costs are from quadratic type we can identify the following marginal effects $k \in \{i,j\};\ i \neq j \in \{1,2\}$:

$$\frac{\partial p_i}{\partial c_k} \geq 0;\ \frac{\partial q_i}{\partial c_i} \leq 0;\ \frac{\partial q_i}{\partial c_j} \geq 0;\ \frac{\partial \pi_i}{\partial c_i} \leq 0;\ \frac{\partial \pi_i}{\partial c_j} \geq 0;\ \frac{\partial^2 p_i}{\partial c_k^2} = 0;\ \frac{\partial^2 q_i}{\partial c_k^2} = 0;\ \frac{\partial^2 \pi_i}{\partial c_k^2} \geq 0$$

3.3 Downstream profits with demand shocks

Market demand in the $N=\{1,\ldots,n\}$ downstream markets has a random size $\tilde{\omega}_i = \omega + \tilde{\varepsilon}_i$, where $\tilde{\varepsilon}_i$ is an idiosyncratic shock. The shocks change the density ω of consumers in the setup described in 3.1. Therefore shocks can be interpreted as random variations of the sum of consumers in the N downstream markets. Assume the shocks are identically and independently distributed with mean zero $E[\ \tilde{\varepsilon}_i\ ]=0$. Therefore the random size of the i-th downstream market is identically and independently distributed with mean ω or $E[\ \tilde{\omega}_i\ ]=\omega$ respectively. This approach is in contrast to the model established in Kranton and Minehart (2000). Without consideration of the market environment they assume each buyer has a random valuation for an input in a buyer-seller network in addition to aggregate shocks on the willingness to pay. Therefore manufactures face idiosyncratic shocks to their demand for inputs.

Example 3.3 (demand shocks): *Assume the existence of idiosyncratic demand shocks and that the transportation cost function is linear or quadratic. For the marginal consumer, the demand of each firm, prices and profits holds:* $c_i = F \cdot e^{\alpha \cdot q_i} \cdot \alpha \quad \Leftrightarrow \quad \psi = 1$ sonst $\psi = 0$; $T(q) := \alpha + \beta q + \gamma q^2$; $d_q := \beta + \gamma$, $0 \leq \alpha, \beta$, $0 < \gamma$; $d_l := \beta$, $0 \leq \alpha$, $0 < \beta$, $0 = \gamma$; $i \neq j \in \{1,2\}$; $d \in \{d_q, d_l\}$:

$$\hat{q} = \frac{p_2 - p_1 + \delta d}{2\delta d};\ q_i = \left(\frac{p_j - p_i + \delta d}{2\delta d}\right)(\omega + \varepsilon_i);\ p_i = \frac{1}{3}c_j + \frac{2}{3}c_i + \delta d;$$

$$\pi_i = \frac{\left(\frac{1}{3}c_j - \frac{1}{3}c_i + \delta d\right)^2}{2\delta d}(\omega + \varepsilon_i) - \psi \cdot Fe^{\alpha\cdot(q_i + \varepsilon_i)}$$

It can be easily checked that comparative static's are the same as in the case without demand shocks. Up to now the model ignores the possibility of different costs situations faced by the firms in consequence of varying demands. Perhaps in case of positive demand shocks inputs can be produced or procured suffering lower costs per unit. Beyond the effect concerning both competitors, perhaps the participation in a vertical network can establish comparative cost advantages for a member of the network. Therefore next section is to specify cost functions explicitly.

4. Efficiency properties and industrial structure

4.1 Efficient industrial structure

In this section we analyze the optimal distribution of realized economies of scale on the upstream level in case of bundling the demand for the input good. Recall that for the upstream level the zero-profit condition holds and therefore all cost savings have to be assigned to the downstream firms. Our purpose is to highlight the optimal pattern of constant prices for the input good that have to be paid by the downstream firms. The examples of chapter 3.2 and 3.3 imply that the profits of the *N* downstream firms are increasing in the market demand. For this reason we explore if it's possible to increase (aggregated) market demand (and therefore to increase aggregated profits) from a network perspective through a proper allocation of input prices to the *N* downstream firms. For simplicity and clearness we investigate and illustrate this important question for linear transportation costs $T(q) := \alpha + \beta q$ in a symmetric setting. The results also emerge for quadratic transportation costs and can be shown for weaker assumptions of symmetry.
Consider two downstream firms ***i,j*** that face Bertrand competition in separated markets as modelled above. Assume both markets are completely identical ex ante. In particular prices for the input good and market prices, demands and profits are the same in equilibrium. Now assume the i-th downstream market is affected by a (positive) demand shock ω and simultaneously the total number of consumers in the j-th downstream market is unchanged ω.

Remark 4.1: *The positive demand shock in the i-th downstream market increases aggregated demand* $q = q_i + q_j$ *and aggregated profits* $\pi = \pi_i + \pi_j$ *from a network* *perspective. Constant input prices independently from the (positive) demand shock yield positive profits for the upstream firm $\pi_U > 0$:*

$$\Delta q = \frac{\frac{1}{3}c_2 - \frac{1}{3}c_1 + \delta d}{2\delta d}\varepsilon;\ \Delta\pi > \frac{\left(\frac{1}{3}c_2 - \frac{1}{3}c_1 + \delta d\right)^2}{2\delta d}\varepsilon;$$

$$\pi_U = \left[\frac{F}{Q}\cdot e^{\alpha\cdot Q} - \frac{F}{Q+\Delta q}\cdot e^{\alpha\cdot(Q+\Delta q)}\right]\left(\omega + \frac{1}{2}\varepsilon\right) > 0$$

When in turn the zero-profit condition holds for the upstream firm, decreasing input prices for at least one downstream firm is an easy implication of positive demand shocks in our setup. In consequence the profit for all intramarginal unit increases. Furthermore additional demand can be served of at least one downstream firm in duopolistic price competition due to the better cost position. How should we design the input prices? Is it possible to increase the profits from network perspective with an additional asymmetric cost shock in presence of a demand shock? Intuitively, one could recommend making the downstream firm *i* (with lager market demand due to a positive demand shock) better off at the expense of the downstream firm *j* with a market demand comparatively smaller. The reason could be seen in the more valuable market shares of downstream firm *i*. If this is true an optimal cost differentiation depending on different market demands would exist. Our results below show that this intuition does not hold in presence of the zero-profit condition for the upstream firm. Questioning optimal cost differentiation does not require explicit consideration of the height of the economies of scale on the upstream level changed due to demand shocks. For simplicity assume identical marginal costs of the competitors in both markets *i,j* after the demand shock such that $c_1 = c_2 = p_i^u = p_j^u > 0$ holds for downstream markets *i,j*. p_i^U, p_j^U denote the prices for the input good in the downstream markets *i,j* needed to guarantee zero-profits on the upstream level. Now assume downstream firm *i* (affected by a positive demand shock) receive a cost reduction of λ_1 what $c_1^i = p_i^U - \lambda_1$ implies. To ensure zero-profits on the upstream level downstream firm *j* has to incur additional costs of λ_2 such that $c_1^j = p_j^U + \lambda_2$ hold.

Proposition 4.1 (1): *Asymmetric costs $c_1^i = p_i^U - \lambda_1$; $c_1^j = p_j^U + \lambda_2$ for downstream firms increase demand and profits from network perspective if and only if $\lambda_2 < \lambda_1 + \lambda_1 \cdot \frac{\varepsilon}{\omega}$ holds.*

Proof: $c_1^i = p_i^U - \lambda_1$; $c_2^i = c_2 \Rightarrow \Delta D_1^1 = \frac{\lambda_1(\omega+\varepsilon)}{6\delta d}$; $c_1^j = p_j^U + \lambda_2$; $c_2^j = c_2$

$$\Rightarrow \Delta D_1^2 = -\frac{\lambda_2\omega}{6\delta d};\ \Delta D = \Delta D_1^1 + \Delta D_1^2 = \frac{1}{6\delta d}\left(\lambda_1\omega + \lambda_1\varepsilon - \lambda_2\omega\right) \Rightarrow\ >0$$

with $\lambda_2 < \lambda_1\left(1+\frac{\varepsilon}{\omega}\right)$; ≤ 0 with $\lambda_2 \geq \lambda_1\left(1+\frac{\varepsilon}{\omega}\right)$ □

Note that it's possible to increase aggregated profits through cost reallocation if constellation of parameters $\lambda_2 < \lambda_1\left(1+\frac{\varepsilon}{\omega}\right)$ does not harm the zero-profit condition for the upstream firm.

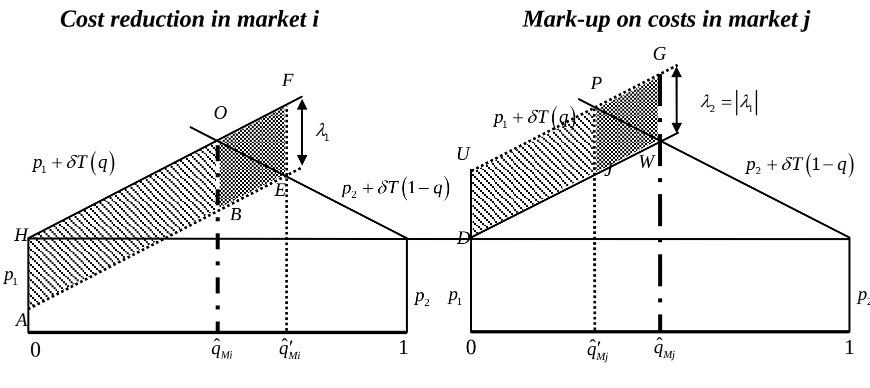
Proposition 4.1 (2): *If the zero-profit condition on the upstream level holds it is impossible to increase demand and profits through cost reallocation from network perspective.*

Proof: $$0=\left(c_1^i-\lambda_1-p_i^U\right)\overbrace{\left(\frac{\lambda_1}{6\delta d}+\frac{1}{2}\right)\left(\omega+\varepsilon\right)}^{D_1^1}+\left(c_1^j+\lambda_2-p_j^U\right)\overbrace{\left(\frac{1}{2}-\frac{\lambda_2}{6\delta d}\right)\omega}^{D_1^2} \Leftrightarrow$$

$$\lambda_2=\left(\lambda_1+\lambda_1\frac{\varepsilon}{\omega}\right)\cdot\left(\delta d+\frac{1}{3}\lambda_1\right)\Big/\left(\delta d-\frac{1}{3}\lambda_2\right) \Rightarrow \lambda_2>\lambda_1\left(1+\frac{\varepsilon}{\omega}\right) \square$$

Note that in case of linear or quadratic transportation costs asymmetric cost allocations don't maximize profits from network perspective. If we make a downstream firm better off through cost reduction of λ_1 this cost advantage also holds for all intramarginal units. Due to the (positive) demand shock in downstream market *i* λ_2 hold for comparatively fewer units. To satisfy the zero-profit condition on the upstream level we therefore have to establish additional costs in shape of λ_2 such that $\lambda_2>|\lambda_1|$ holds. The gain of demand in market *i* is overcompensated through the loss of demand in market *j*. Therefore in presence of asymmetric prices for the input good, demand and profit decreases from network perspective. The figure below shows that this result arises also for $\varepsilon=0$.

Figure 2: cost situation



Decreasing procurement costs in market *i* cause losses for the upstream firm for all intramarginal units of the input good demanded by downstream firm *i* before cost reduction. This is illustrated by the area ABOH. In consequence of the lower procurement costs downstream firm *i* choose a lower optimal price and serves therefore additional demand $\hat{q}'_{Mi} - \hat{q}_{Mi}$ (See for a more general proof remark 3.1). Establishment of additional costs for the input good in market *j* such that $\lambda_2 = |\lambda_1|$ holds implies a decline of served market demand $\hat{q}_{Mj} - \hat{q}'_{Mj}$ by downstream firm *j*. Although changes in demand compensate each other the upstream firm incurs losses illustrated by the area BEFO. This harms the zero-profit condition for the upstream firm. Additional revenues in market *j* expressed by area DJPU are lower than missing revenues in market *i* represented by area AEFH. Therefore the condition of proposition 4.1 (1) can not be satisfied. In the next section we explore implications of demand shocks on negotiations about prices of inputs between the downstream firms in presence of the upstream zero-profit condition. We also discuss the relation of our results to the propositions stated above.

4.2 Strategic firms and industrial structure

In presence of (idiosyncratic) demand shocks the contribution of the *N* downstream firms to the economies of scale realized during production of the input good on the upstream level and gained through bundling demand for the input good differs. Assume that the downstream firms negotiate about the division of realized cost savings and the procurement prices for inputs respectively. Possible outcomes of this negotiations and the correspondence to the efficiency results derived in chapter 4.1 arise as important questions. Therefore we analyze an n-person bargaining game with the player set $N = \{1, \ldots, n\}$ consisting of the downstream firms. First of all we have to determine cooperation surplus and individual outside options of the *n* firms on the downstream level.
The rent to be divided $R: (0,\infty) \to \mathbb{R}_0^+$; $Q \mapsto R(Q)$ can be calculated as the difference between the market price for the input good and the average costs occurring on the upstream level in case of producing the input good bundled. Furthermore this difference is multiplied by the aggregated market demand from network perspective to determine absolute cooperation surplus:

$$R(Q) := \left(p_M - \frac{F}{Q} \cdot e^{\alpha \cdot Q} \right) \cdot Q \quad \text{with} \quad Q = \sum_{i=1}^{n} q_i \, ; \qquad \frac{\partial R}{\partial q_i} > 0$$

Note that in our setup a downstream firm is able to procure input goods by own production. Therefore in case of no cooperation the outside option $A_i: (0,\infty) \to \mathbb{R}_0^+$; $q_i \mapsto A_i(q_i)$ of the i-th downstream firm can be calculated as difference between market price for the input good and the average costs of own production:

$$A_i(q_i) := \left(p_M - \frac{F}{q_i} \cdot e^{\alpha \cdot q_i} \right) \cdot q_i \quad \text{with} \quad i \in N \, ; \qquad \frac{\partial A}{\partial q_i} > 0$$

To define the downstream firms bargaining power depending on the special demand for the input good *G* und therefore individual contribution to realised cost savings the Shapley revenue rule is applied. The cooperation surplus *R* is allocated according to this rule. The Shapley value is calculated as the weighted average of the downstream firm's marginal contribution to all possible coalitions of firms. The Shapley value is a standard way to define bargaining power in groups and is applied in research on supply structures (Hart and Moore 1990; Kranton and Minehart 2000). The Shapley value φ_i for downstream firm *i* is determined as:

$$\varphi_i(v) = \sum_{\substack{K \subseteq N \\ i \in K}} \frac{(k-1)!(n-k)!}{n!}\left[v(K) - v(K \setminus \{i\})\right], \; i \in N, \; k := |K|, \; n := |N| \text{ with:}$$

$$\upsilon : 2^N \to \mathbb{R}, \; \upsilon(K \subseteq N) := \left(p_M - \frac{F}{\sum_{j \in K} q_j} e^{\alpha \sum_{j \in K} q_j} \right) \sum_{j \in K} q_j; \; \upsilon(\varnothing) := 0; \sum_{i=1}^{n} \varphi_i(v) = R$$

The worth function $\upsilon : 2^N \to \mathbb{R}$ describes the total expected gain from the cooperation $K \subseteq N$, independent of what the actors outside of *K* do. Alternatively φ_i can be expressed as:

$$\varphi_i(v) =$$

$$\sum_{\substack{K \subseteq N \\ i \in K}} \frac{(k-1)!(n-k)!}{n!} \left[\left(p_M - \frac{F}{\sum_{i \neq l} q_l + q_i} e^{\alpha\left(\sum_{i \neq l} q_l + q_i\right)} \right) \sum_{\substack{i \neq l \\ i \in K}} q_l - \left(p_M - \frac{F}{\sum_{i \neq l} q_l} e^{\alpha \sum_{i \neq l} q_l} \right) \sum_{i \neq l} q_l \right]$$

Proposition 4.2 (1) (Comparative static's): *The Shapley value is strictly increasing in the own and strictly decreasing in the other downstream firm's demand for the input good G:*

$$\frac{\partial \varphi_i(v)}{\partial q_i} > 0; \qquad \frac{\partial \varphi_j(v)}{\partial q_i} < 0 \qquad j \neq i$$

Therefore the Shapely revenue rule makes parties better off with comparatively valuable alternatives. Consequently parties with comparatively less valuable alternatives receive less then the n-th share of cooperation surplus. In our model downstream firms realizing the highest positive (idiosyncratic) demand shocks dispose of relatively valuable outside options. For reasons of clearness this derivation does not consider that parties calculate cooperation surplus and outside options in anticipation of the result of the bargaining game. Therefore no perfect foresight on the implications of bargaining results in particular on costs, prices und resulting demand is assumed.

Now the negotiated shares of *R* can be calculated as individual input prices per unit p_i^u that have to be paid by the *N* downstream firms respectively. Calculation has to take into consideration the zero-profit condition for the upstream firm.

$$\varphi_i = \left(p_M - p_i^u\right) q_i \Leftrightarrow p_i^u = p_M - \frac{\varphi_i}{q_i}, \; \varphi_j = \left(p_M - p_j^u\right) q_j \Leftrightarrow p_j^u = p_M - \frac{\varphi_j}{q_j}; \; \frac{\partial p_j^u}{\partial q_i} > 0$$

Proposition 4.2 (2): *Assume that downstream firm i disposes of a complementarily valuable outside option in consequence of idiosyncratic demand shocks. The Shapely value leads to heterogeneous prices for the input good in the network.*
Proof: See the Appendix I

Note that this asymmetric distribution of procurement costs among the downstream firms is in contrast to the efficiency results of chapter 4.1. In particular the outcome of the Shapely revenue rule of this n-person bargaining game does not maximize aggregate profits from network perspective. It is easily checked that the shape of the worth function $\upsilon(\,.\,)$ is not convex. Therefore a priori it is not possible to determine stability of the network in presence of idiosyncratic demand shocks. However stability can be proven.

Proposition 4.2 (3) (stability): *The Shapely value as bargaining solution in the multilateral case implies stability of the network. The network is stable independently from concrete realisation of the idiosyncratic demand shocks. Firms can never be better off with own production for example in case of strong positive demand shocks. In particular* $\varphi_i(\upsilon)-A_i>0\ \ \forall q_i$ *is satisfied.*
Proof: See the Appendix II

The result is intuitive and implied by the superadditivity property of the worth function. Comparatively small demands for the input good *G* increase the cooperation surplus too. From proposition 4.1 (2) together with proposition 4.2 (3) follows theorem 4.2:

Theorem 4.2: *In case of idiosyncratic demand shocks negotiations on the prices for the input good leads to a stable but inefficient (complete) network.*

Recall the assumption that maximum willingness to pay $\overline{\upsilon} \in \mathbb{R}^+$ is high enough that every individual buys in equilibrium in the *N* downstream markets. Therefore we don't analyze questions relating to overall social welfare.

Seen together, the results obtained for the network as defined above yield a number of observations. First we note that firms generally have an incentive to collaborate in shape of bundling the demand for an input good, so the empty network is never incentive compatible. Second, (idiosyncratic) demand shocks lead to an asymmetric distribution of bargaining power into negotiations on input prices between the downstream firms. Third, this difference in firms outside options does not threat the stability of the network but has negative consequences for efficiency from network perspective. Individual considerations lead firms to a distribution of input prices that does not maximize aggregated profits. This problem becomes more relevant if the assumption $q_i<1/\alpha\,;\,(q_1+\ldots+q_n)<1/\alpha\ \forall q_i$ is softened. Then it could feasibly happen that in presence of strong demand shocks the bargaining solution leads to instability of the network (Rotemberg and Saloner 1986). In the next section we briefly discuss some possibilities to soften or solve this kind of problems.

4.3 Additional comments

In our approach efficiency problems of the Nash cooperative bargaining solution arise because downstream firms realizing comparatively large positive demand shocks doesn’t internalise the external effect of their strong bargaining power due valuable outside options. In particular they don’t consider demand effects of higher input prices for downstream firms confronted by a lower density of consumers. In consequence inefficiencies arise from network perspective. Therefore solution concepts have to take into consideration possibilities of internalising these extern effects. On the one hand we could establish a system of side payments inspired by tax and transfer systems.

Starting from the expected market demand ω additional demand could be taxed per unit. This implies increasing procurement costs of the input good for all units exceeding expected market demand ω. These tax revenues could be used to subsidize the weaker downstream parties. Note that the expected realization of the demand shock is zero for all firms ex ante. Therefore all downstream firms would agree with these non linear prices ex ante. In this case the question arises how strong are incentives to deviate from ex ante agreement and to renegotiate the input prices ex post. This question becomes much more interesting if we generalize our setup. In our paper the ex ante agreement is renegotiation proof because in case of cooperation all downstream firms reach at least the same costs that would occur in case of own production which in turn are never higher then network extern market prices for the input good. If the downstream firms are not able to write complete conditional contracts from ex ante perspective perhaps a proper structure of control- and governance mechanisms can be implemented to soften the described problem of externalities. However these possibilities have to be analyzed carefully. For example the vertical control problem inherent in delegation is essentially that of double marginalization of rents. Furthermore establishment of a governance structure regularly implies that the zero-profit condition and the implicitly assumed productive efficiency on upstream level vanish. Therefore additional questions related to problems of delegation- and incentive constraints arise (Mookherjee 2006).

5. Concluding Remarks

Our goal in this paper was to explore the existence of a trade-off between profitability and stability in vertical networks in presence of demand uncertainty. For the chosen setup we showed bargaining on cooperation surplus leads to inefficient allocations from network perspective. In case of weaker assumptions with respect to the outside option or other revenue rules stability of cooperation and network is expected to be endangered. Some solution concepts are shortly introduced and discussed in the section above. Our results may suggest at least two avenues for future research. First, to analyze proper governance structures of vertical networks the relation of this paper to the literature of incomplete contracts in particular to the literature of the theory of the firm has to be investigated carefully. Secondly, implications of heterogeneity among firms in vertical networks could be studied from a more practical point.

Varying demand among the downstream firms could be the result of different business strategies. In this case heterogeneity would be endogenous and incentive mechanisms could be studied from network perspective.

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Appendix I: Proof of Proposition 4.2 (2)

Assume the *n* markets completely identical ex ante. Zero profit condition implies:

$$p_1 q_1 + p_2 q_2 + ... + p_n q_n \overset{!}{=} F \cdot e^{\alpha(q_1 + q_2 + ... + q_n)} \quad \Leftrightarrow \quad \frac{\sum_{i=1}^{n} p_i q_i}{Q} \overset{!}{=} \frac{F}{Q} \cdot e^{\alpha \cdot Q}$$

Consider a positive demand shock ε_i in downstream market i and remember assumptions of chapter three:

$$\frac{F}{q_1 + ... + q_i + \varepsilon_i ... + q_n} \cdot e^{\alpha \cdot (q_1 + ... + q_i + \varepsilon_i ... + q_n)} < \frac{F}{q_1 + q_2 + ... + q_n} \cdot e^{\alpha \cdot (q_1 + q_2 + ... + q_n)}$$

Therefore decreasing average production costs on the upstream level are induced. Note $\partial p_j / \partial q_i > 0$ holds for at least one downstream firm. A homogenous mark up Δ on input prices would lead to increasing average revenues for the upstream firm what contradicts the zero profit condition. For that reason demand shocks lead to heterogeneous input prices.□

Appendix II: Proof of Proposition 4.2 (3)

$$A_i(q_i) := \left(p_M - \frac{F}{q_i} \cdot e^{\alpha \cdot q_i} \right) \cdot q_i$$

$$\varphi_i(v) =$$

$$\sum_{\substack{K \subseteq N \\ i \in K}} \frac{(k-1)!(n-k)!}{n!} \left[\left(p_M - \frac{F}{\sum_{i \neq l} q_l + q_i} e^{\alpha\left(\sum_{i \neq l} q_l + q_i\right)} \right) \sum_{\substack{i \neq l \\ i \in K}} q_l - \left(p_M - \frac{F}{\sum_{i \neq l} q_l} e^{\alpha \sum_{i \neq l} q_l} \right) \sum_{i \neq l} q_l \right]$$

Note: $\sum_{\substack{K \subseteq N \\ i \in K}} \frac{(k-1)!(n-k)!}{n!} = 1$

$$\varphi_i - A_i = \sum_{\substack{K \subseteq N \\ i \in K}} \frac{(k-1)!(n-k)!}{n!} \left(p_M - \frac{F}{\sum_{i \neq l} q_l + q_i} e^{\alpha\left(\sum_{i \neq l} q_l + q_i\right)} \right) \left(\sum_{i \neq l} q_l + q_i \right) -$$

$$\sum_{\substack{K \subseteq N \\ i \in K}} \frac{(k-1)!(n-k)!}{n!} \left[\left(p_M - \frac{F}{\sum_{i \neq l} q_l} e^{\alpha \sum_{i \neq l} q_l} \right) \sum_{i \neq l} q_l - \left(p_M - \frac{F}{q_i} e^{\alpha \cdot q_i} \right) q_i \right]$$

$$\left(p_M - \frac{F}{\sum_{i\neq l} q_l + q_i} e^{\alpha\left(\sum_{i\neq l} q_l + q_i\right)}\right)\left(\sum_{i\neq l} q_l + q_i\right) - \left(p_M - \frac{F}{\sum_{i\neq l} q_l} e^{\alpha \sum_{i\neq l} q_l}\right)\sum_{i\neq l} q_l - \left(p_M - \frac{F}{q_i}\cdot e^{\alpha\cdot q_i}\right)\cdot q_i$$

$$= \underbrace{\left(p_M - \frac{F}{\sum_{i\neq l} q_l + q_i} e^{\alpha\left(\sum_{i\neq l} q_l + q_i\right)}\right)\sum_{i\neq l} q_l - \left(p_M - \frac{F}{\sum_{i\neq l} q_l} e^{\alpha \sum_{i\neq l} q_l}\right)\sum_{i\neq l} q_l}_{>0} \quad +$$

$$\underbrace{\left(p_M - \frac{F}{\sum_{i\neq l} q_l + q_i} e^{\alpha\left(\sum_{i\neq l} q_l + q_i\right)}\right) q_i - \left(p_M - \frac{F}{q_i}\cdot e^{\alpha\cdot q_i}\right)\cdot q_i}_{>0}$$

$$\Rightarrow \ \varphi_i - A_i > 0$$

□