The comparative static results for a *NARP*-maximising processing cooperative in short-run equilibrium

Arvid Senhaji[1]

Abstract

The adjustment of a *NARP-maximising* processing cooperative facing an aggregated raw agricultural supply schedule with a positive slope due to decreasing returns to scale is revisited. When the peak coordinator being responsible for the daily management of the processing cooperative is able to control members' supplies of the raw agricultural product, unambiguous qualitative comparative static results can be derived on the basis of the unrestricted NARP function proven to be convex in final output price and processing production factor prices. In general, no unambiguous qualitative comparative static results are forthcoming on the basis of the NARP function when the peak coordinator is unable to restrict members' supplies of the raw agricultural product.

Key words:
Firm behaviour, processing cooperatives, duality theory.

1. Introduction

The analysis of agribusiness processing cooperatives experienced a breakthrough in the middle of the 1950`s when agricultural economists started to adapt the

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[1] Arvid Senhaji is an adviser at the Norwegian Royal Ministry of Government Administration and Reform, Department of Competition Policy, Arvid.Senhaji@fad.dep.no. The Norwegian Research Council financed this research in terms of a P.h.D scholarship provided to the author by the Norwegian Agricultural Economics Research Institute. Financial support by the Royal Ministry of Government Administration and Reform is kindly acknowledged. The author wants to thank Petter Frenger, Karl-Ove Moene, Erik Biørn, Steinar Strøm, Ådne Cappelen, and Gunnar Birkelund for helpful comments. The usual disclaimers apply.

«neoclassical» theory of the firm in order to investigate the adjustment of different types of agribusiness cooperatives operating in a perfectly competitive market environment without uncertainty. (Sexton et al. 1989) and (Boyle 1998, 2004) extended the «neoclassical» theory of processing cooperatives further by developing a duality methodology based on (McFadden 1978)'s restricted profit function. (Senhaji 2007) proves that the duality approach of (Sexton et al. 1989) and (Boyle 1998, 2004) is only correct when the aggregated raw agricultural supply schedule facing the processing cooperative is completely vertical. In this paper I revisit the short-run analysis of a processing cooperative facing an aggregated raw agricultural supply schedule that is not completely vertical, but instead exhibits a general positive slope.

The number of cooperative members is fixed, and members are presumed to be able to reach an agreement saying that business shall be managed so that the net average revenue product (*NARP*) is maximised. If the peak coordinator being responsible for the daily management of the processing cooperative is able to control members' supplies of the raw agricultural product and secure them the maximum *NARP* achievable, qualitative comparative static results can be derived on the basis of the unrestricted NARP function proven to be convex in final output price and processing production factor prices.

In general, no unambiguous qualitative comparative static results for the relative choice functions are forthcoming on the basis of the equilibrium NARP function when the peak coordinator is unable to restrict members' supplies of the raw agricultural product.

2. The conceptual framework and the main assumptions

In this paper I focus on the adjustment of a regional processing cooperative that typically are established by farmers in order to avoid being exploited by profit-maximising investor-owned firms (IOFs) with market power. The processing cooperative only collects and processes the raw agricultural produce supplied by the fixed number of members equal to n^c. Cooperative farmers maximise profits and operate as price takers in all markets. Thus, the processing cooperative faces an aggregated raw agricultural supply schedule comprising the n^c members' profit-maximising supply functions. Whenever the cooperative's financial structure is patronage based, the aggregated raw agricultural supply schedule will be a function of the total payment that members of the cooperative society receive per unit of the raw agricultural product equal to *NARP*, together with *D* strictly positive farming production factor prices in the vector $\mathbf{w^d}$, and a policy support index denoted by T_a:

$$x_a^S = \mathrm{S_a^C}\left(NARP, \mathbf{w^d}; T_a\right) = \sum_{i=1}^{n^c} \mathrm{S_i^a}(NARP, \mathbf{w^d}; T_a). \tag{2.1}$$

A peak coordinator elected by the n^c members is responsible for the daily management of the processing and marketing business. The n^c members collectively constitute a «principal» and are presumed able to reach an agreement saying that the peak coordinator must run the processing business so as to maximise the *NARP* paid to members per unit of the raw agricultural product.

A processing cooperative that is able to restrict members' supplies of the raw agricultural product is distinguished from a processing cooperative that is not able to do so. The former type of processing cooperatives is labelled a restricting processing cooperative while the latter is labelled an unrestricting processing cooperative. All net return is rebated to members according to patronage defined as a farmer's share of the aggregated supply of the raw agricultural product. In the subsequent analysis I do not consider retaining of funds.

Let the production process of the cooperative processing firm be described by the production function

$$y = \mathrm{f}(x_1^b,\ldots,x_B^b,x_a^C,K) = \mathrm{f}(\mathbf{x^b},x_a^C,K) = \mathrm{f}(\mathbf{x},K). \quad (2.2)$$

In expression (2.2) y denotes quantity produced of the final output, K is the fixed amount of productive capital, and x_j^b is the j$^{\text{th}}$ processing production factor in the processing input vector $\mathbf{x^b}$. Labour, fuels, energy, and non-agricultural materials are examples of processing inputs with a corresponding processing input price vector equal to $\mathbf{w^b} = (w_1^b,\ldots, w_B^b)$ containing B strictly positive processing factor prices. The vector $\mathbf{x}$ includes all inputs given by the B processing factors together with the raw agricultural quantity x_a^C. The production function is assumed to be continuous from above, and to exhibit weak monotonicity in the input vector $\mathbf{x}$. The raw agricultural quantity x_a^C is a strictly essential production factor. The strictly positive final output price equals P, and the cooperative processing firm is a price taker in the output market as well as in all processing production factor markets. Let F denote fixed cost. In order to alleviate notational clutter I will not include the fixed amount of capital as an argument in the short-run equilibrium choice functions analysed in the subsequent chapters.

The relationship between the gross revenue product (*GRP*) defined in expression (2.3) as total revenue minus variable processing cost, and the net revenue product (*NRP*), the net average revenue product (*NARP*), and the average revenue product (*ARP)* are as follows:

$$GRP = \left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b\right)\right), \quad (2.3)$$

$$NRP = \left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b\right) - F\right) = GRP - F, \quad (2.4)$$

$$NARP = \left(\frac{\left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b\right) - F\right)}{x_a^{\mathrm{C}}}\right) = \left(\frac{(GRP - F)}{x_a^{\mathrm{C}}}\right), \text{ and} \quad (2.5)$$

$$ARP = \left(\frac{\left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) \right)}{x_a^{\mathrm{C}}} \right) = \left(\frac{(GRP)}{x_a^{\mathrm{C}}} \right). \qquad (2.6)$$

With these definitions made, I am ready to derive the set of qualitative comparative static results for a processing cooperative maximising *NARP* while facing an upward sloping aggregated raw agricultural supply schedule due to decreasing returns to scale in agriculture. Numerous other objectives that a processing cooperative could pursue are described in (LeVay 1983) and (Sexton 1984), among others.

3. The unrestricted NARP-maximising behavioural functions

In order to derive the unrestricted NARP-maximising choice functions denoted by superscript «U», I first present the maximisation problem facing the peak coordinator that is now assumed to be able to restrict members' supplies in order to secure them the maximum *NARP* achievable:

$$\max_{y,\mathbf{x^b},x_a^C} \{NARP\} = \max_{y,\mathbf{x^b},x_a^C} \left\{ \left(\frac{Py - \left(\sum_{j=1}^{B} w_j^b x_j^b \right) - F}{x_a^C} \right) \mid \mathrm{f}(\mathbf{x^b}, x_a^C) \geq y \right\}. \qquad (3.1)$$

Without loss of generality I divide the maximisation problem facing the peak coordinator of the restricting processing cooperative into two separate stages. On stage one the peak coordinator treats the raw agricultural quantity x_a^C as an exogenous variable equal to x_0^a. As suggested by (Sexton et al. 1989: 56), and (Boyle 1998, 2004), maximising *NARP* with regards to the processing production factor vector $\mathbf{x^b}$ is then equivalent to maximising *GRP* with regards to $\mathbf{x^b}$:

$$\max_{y,\mathbf{x^b},x_a^C} \{NARP \mid x_a^C = x_0^a\} \Leftrightarrow \max_{y,\mathbf{x^b}} \{GRP \mid x_a^C = x_0^a\}. \qquad (3.2)$$

The restricted variable processing cost function $\mathrm{V^R}(\mathbf{w^b}, y, x_0^{\ a})$ and the restricted GRP function $\mathrm{GRP^R}(P, \mathbf{w^b}, x_0^a)$ are defined directly by the following maximisation problems:

$$\mathrm{V^R}\left(\mathbf{w}^{\mathrm{b}}, y, x_0^a\right) = \min_{\mathbf{x^b}} \left\{ \left(\sum_{j=1}^{B} w_j^b x_j^b \right) \mid \mathrm{f}\left(\mathbf{x^b}, \mathrm{x}_0^a\right) \geq y \right\}, \text{ and} \qquad (3.3)$$

$$\mathrm{GRP^R}\left(P, \mathbf{w^b}, x_0^a\right) = \max_y \left\{ Py - \mathrm{V^R}\left(\mathbf{w^b}, y, x_0^a\right) \right\}. \qquad (3.4)$$

The restricted variable processing cost function $V^R(\mathbf{w^b}, y, x_0^a)$ in expression (3.3) is the support function of the implicit processing input requirement set $L^*(y, x_0^a)$[2] defined as:

$$L^*\left(y, x_0^a\right) = \left\{\mathbf{x^b} : \left(\sum_{j=1}^{B} w_j^b x_j^b\right) \geq \mathrm{V}^{\mathrm{R}}\left(\mathbf{w^b}, y, x_0^a\right) \text{ for all } \mathbf{w^b} > \mathbf{0_B}.\right\} \tag{3.5}$$

Likewise, the restricted GRP function in expression (3.4) is the support function of the implicit production possibilities set $T^*(x_0^a)$[3] defined as:

$$T^*\left(x_0^a\right) = \left\{\begin{array}{l}\left(y\ \mathbf{x^b}\right) : \left(Py - \left(\sum_{j=1}^{B} w_j^b x_j^b\right)\right) \leq \mathrm{GRP}^{\mathrm{R}}\left(P, \mathbf{w^b}, x_0^a\right) \\ \text{for all } \mathbf{w^b} > \mathbf{0_B} \text{ and } x_0^a > 0.\end{array}\right\} \tag{3.6}$$

The restricted NARP function defined in expression (3.7) specifies the maximum price the co-op can pay for the given amount of the raw agricultural product x_0^a after processing cost and fixed cost are paid:

$$\mathrm{NARP}^{\mathrm{R}}\left(P, \mathbf{w^b}, x_0^a, F\right) = \left(\frac{\mathrm{GRP}^{\mathrm{R}}\left(P, \mathbf{w^b}, x_0^a\right) - F}{x_0^a}\right). \tag{3.7}$$

(Senhaji 2007) derives the following six properties for the restricted NARP function:

1. $\mathrm{NARP}^R(P, \mathbf{w^b}, x_0^a, F) \geq (-F / x_0^a)$;
2. If $P_2 \geq P_1$, then $\mathrm{NARP}^R(P_2, \mathbf{w^b}, x_0^a, F) \geq \mathrm{NARP}^R(P_1, \mathbf{w^b}, x_0^a, F)$;
3. If $\mathbf{w_2^b} \geq \mathbf{w_1^b}$ [4], then $\mathrm{NARP}^R(P, \mathbf{w_2^b}, x_0^a, F) \leq \mathrm{NARP}^R(P, \mathbf{w_1^b}, x_0^a, F)$;

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[2] $L^*(y, x_0^a)$ is either identical or a monotonised convexification of the original restricted input requirement set given by $L\left(y, x_0^a\right) = \left\{\mathbf{x^b} : \mathrm{f}\left(\mathbf{x^b}, x_0^a\right) \geq y, \mathbf{x^b} > \mathbf{0_B}, x_0^a > 0\right\}$ whenever $L(y, x_0^a)$ contains nonconvex configurations that are never utilised by a rationale cost-minimising processing firm. See (Chambers 1994: 86) for a further discussion of this point.

[3] $T^*(x_0^a)$ is either identical or a monotonised convexification of the original restricted production possibilities set given by $T\left(x_0^a\right) = \left\{\mathbf{x^b} : \mathrm{f}\left(\mathbf{x^b}, x_0^a\right) \geq y, x_0^a > 0\right\}$ whenever $T(x_0^a)$ contains nonconvex configurations that are never utilised by a rationale *NARP*-maximising processing firm.

[4] Vector inequalities follow the subsequent convention throughout the paper: $\mathbf{w_2^b} > \mathbf{w_1^b}$ means that every element of $\mathbf{w_2^b}$ is strictly greater than the corresponding element of $\mathbf{w_1^b}$; $\mathbf{w_2^b} \geq \mathbf{w_1^b}$ means that every element of $\mathbf{w_2^b}$ is at least as large as

4. $\text{NARP}^{\text{R}}(P, \mathbf{w^b}, x_0{}^a, F)$ is convex and continuous[5] in $(P, \mathbf{w^b})$;

5. $\text{NARP}^{\text{R}}(\text{t}P, \text{t}\mathbf{w^b}, \text{x}_0{}^a, \text{t}F) = \text{t NARP}^{\text{R}}(P, \mathbf{w^b}, \text{x}_0{}^a, F)$, where «t» is a positive scalar; and finally

6. If $\text{GRP}^{\text{R}}(P, \mathbf{w^b}, x_0{}^a)$ is differentiable in the price vector $(P, \mathbf{w^b})$, $\text{NARP}^{\text{R}}(P, \mathbf{w^b}, x_0{}^a, F)$ is also differentiable in these (B+1) strictly positive prices since the latter function is a positive linear transformation of the former. The gradient of the restricted NARP function in the price vector $(P, \mathbf{w^b})$ is equal to (*Hotelling-McFadden's lemma*):

$$\nabla_{(P,\mathbf{w^b})} \text{NARP}^{\text{R}}\left(P,\mathbf{w^b},x_0^a,F\right)=\left(\left(\frac{\text{S}^{\text{R}}(\cdot)}{x_0^a}\right) \quad \ldots \quad -\left(\frac{\text{D}_{\text{B}}^{\text{R}}(\cdot)}{x_0^a}\right)\right), \text{ and} \qquad (3.8)$$

$$\left(\frac{\nabla_{(P,\mathbf{w^b})} \text{NARP}^{\text{R}}(\cdot)}{-\dfrac{\partial \text{NARP}^{\text{R}}(\cdot)}{\partial F}}\right)=\left(\text{S}^{\text{R}}\left(P,\mathbf{w^b},x_0^a\right) \quad \ldots \quad -\text{D}_{\text{B}}^{\text{R}}\left(P,\mathbf{w^b},x_0^a\right)\right) \Leftrightarrow \qquad (3.9)$$

$$\left(\frac{\nabla_{(P,\mathbf{w^b})} \text{NARP}^{\text{R}}\left(P,\mathbf{w^b},x_0^a,F\right)}{-\dfrac{\partial \text{NARP}^{\text{R}}\left(P,\mathbf{w^b},x_0^a,F\right)}{\partial F}}\right)=\nabla_{(P,\mathbf{w^b})} \text{GRP}^{\text{R}}\left(P,\mathbf{w^b},x_0^a\right). \qquad (3.10)$$

$$\text{NARP}\left(P,\mathbf{w^b},x_a^C,F\right)=\left(\frac{\text{GRP}^{\text{R}}\left(P,\mathbf{w^b},x_a^C\right)-F}{x_a^C}\right). \qquad (3.11)$$

On stage two the peak coordinator maximises the NARP function defined in expression (3.11) with regards to the raw agricultural quantity $x_a{}^C$:

$$\text{NARP}^{\text{U}}\left(P,\mathbf{w^b},F\right)=\max_{x_a^C}\left\{\text{NARP}\left(P,\mathbf{w^b},x_a^C,F\right)\right\}. \qquad (3.12)$$

The first- and second-order conditions for this maximisation problem read:

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the corresponding element of $\mathbf{w_1^b}$ and that at least one element of $\mathbf{w_2^b}$ is strictly greater than the corresponding element in $\mathbf{w_1^b}$.

[5] Furthermore, the restricted NRP- and NARP function are both convex and continuous in the extended vector $(P, \mathbf{w^b}, F)$.

$$\left(\frac{\partial \text{NARP}\left(P,\mathbf{w^b},x_a^C,F\right)}{\partial x_a^C}\right)=\left(\frac{\text{NMRP}(\cdot)-\text{NARP}(\cdot)}{x_a^C}\right)=0, \text{ and} \tag{3.13}$$

$$\left(\frac{\partial^2 \text{NARP}\left(P,\mathbf{w^b},x_a^C,F\right)}{\partial x_a^{C2}}\right)=\left(\frac{\partial \text{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right)}{\partial x_a^C}\right)\left(\frac{1}{x_a^C}\right)<0. \tag{3.14}$$

The net marginal revenue product NMRP(P, $\mathbf{w^b}$, $x_a{}^C$, K) in expression (3.13) measures the increase in *GRP* and *NRP* from processing an additional unit of the raw agricultural product, and is identical to the raw agricultural shadow price:

$$\left(\frac{\partial \text{GRP}\left(P,\mathbf{w^b},x_a^C\right)}{\partial x_a^C}\right)=\left(\frac{\partial \text{NRP}(\cdot)}{\partial x_a^C}\right)=\text{NMRP}\left(P,\mathbf{w^b},x_a^C,K\right). \tag{3.15}$$

When the raw agricultural shadow price is declining in the aggregated raw agricultural quantity $x_a{}^C$, the second-order condition in expression (3.14) is fulfilled. The optimal demand of the raw agricultural product for the restricting processing cooperative is implicitly defined by the first-order condition in expression (3.13) as a function of final output price P, the processing production factor prices $\mathbf{w^b}$, and fixed cost F:

$$x_a^U=\text{x}_\text{a}^\text{U}\left(P,\mathbf{w^b},F\right). \tag{3.16}$$

Notice the important fact that the optimal raw agricultural demand function defined in expression (3.16) is not a function of the input prices in the vector $\mathbf{w^d}$. But this result is contingent on the fact that the aggregated raw agricultural supply schedule is sufficiently large to allow for the realisation of the maximum *NARP* in the first place. Thus, it is assumed here that the aggregated inverse raw agricultural supply schedule given by the cooperative farmers aggregated marginal cost function MC_a($\mathbf{w^d}$, $x_a{}^C$; T_a) intersects with the NARP function to the right of or at the latter function's apex as illustrated in figure 1 above.

$$\text{NARP}^\text{U}\left(P,\mathbf{w^b},F\right)=\text{NARP}\left(P,\mathbf{w^b},\text{x}_\text{a}^\text{U}\left(P,\mathbf{w^b},F\right),F\right). \tag{3.17}$$

The properties of the unrestricted NARP function defined in expression (3.12) that enables me to recapture both the relative and ordinary unrestricted NARP-maximising choice functions describing the behaviour of a restricting processing cooperative, are now stated (*III*):

1. $\text{NARP}^\text{U}(P, \mathbf{w^b}, F) \geq (-F / (\text{x}_\text{a}{}^\text{U}(P, \mathbf{w^b}, F))$;

2. if $P_2 \geq P_1$, then $\text{NARP}^\text{U}(P_2, \mathbf{w^b}, F) \geq \text{NARP}^\text{U}(P_1, \mathbf{w^b}, F)$;

3. if $\mathbf{w_2^b} \geq \mathbf{w_1^b}$, then $\text{NARP}^\text{U}(P, \mathbf{w_2^b}, F) \leq \text{NARP}^\text{U}(P, \mathbf{w_1^b}, F)$;

4. $\text{NARP}^{\text{U}}(P, \mathbf{w^b}, F)$ is positively linearly homogeneous in the extended vector
$(P, \mathbf{w^b}, F)$;

5. $\text{NARP}^{\text{U}}(P, \mathbf{w^b}, F)$ is convex and continuous in the extended vector $(P, \mathbf{w^b}, F)$; and finally

6. if $\text{NARP}^{\text{U}}(P, \mathbf{w^b}, F)$ is differentiable in the extended vector $(P, \mathbf{w^b}, F)$, the gradient of $\text{NARP}^{\text{U}}(P, \mathbf{w^b}, F)$ in $(P, \mathbf{w^b})$ is identical to (*The Viner-Wong envelope theorem*):

$$\nabla_{(P,\mathbf{w^b})}\text{NARP}^{\text{U}}\left(P,\mathbf{w^b},F\right) = \left(\left(\frac{\text{S}^{\text{U}}(\cdot)}{\text{x}_{\text{a}}^{\text{U}}(\cdot)}\right) \quad \ldots \quad -\left(\frac{\text{D}_B^{\text{U}}(\cdot)}{\text{x}_{\text{a}}^{\text{U}}(\cdot)}\right)\right) \qquad (3.18)$$

$$\left(\frac{\nabla_{(P,\mathbf{w^b})}\text{NARP}^{\text{U}}(\cdot)}{-\left(\dfrac{\partial\text{NARP}^{\text{U}}(\cdot)}{\partial F}\right)}\right) = \left(\text{S}^{\text{U}}\left(P,\mathbf{w^b},F\right) \ldots - \text{D}_{\text{B}}^{\text{U}}\left(P,\mathbf{w^b},F\right)\right). \qquad (3.19)$$

Proofs of statements (*III*) are found in appendix 1. The crux of the short-run neo-classical theory for a restricting processing cooperative firm is presented in the following formal theorem:

THEOREM 1: *The qualitative comparative static results of a restricting processing cooperative confronting an upward sloping raw agricultural supply function and whose goal is to maximise the net average revenue product, can be summarised in the statement that the matrix of cross-partial derivatives of the type* $\left(\dfrac{\partial^2\,\text{NARP}^{\text{U}}\left(P,\mathbf{w^b},F\right)}{\partial Z_i \partial Z_j}\right)$*, where* Z_i*,* Z_j *are any prices in the price vector* $(P, \mathbf{w^b})$*, is symmetric and positive semidefinite.*

The symmetry of the matrix in theorem 1 follows from Young's theorem. This symmetry immediately yields the following reciprocity relations related to the optimal relative final output (y^U/x_a^U) identical to the inverse of the optimal conversion factor, and the B optimal relative processing input demands (x_j^{bU}/x_a^U), j = 1,…, B.
COROLLARY 1.1:

$$\left(\frac{\partial \left(\frac{y^U}{x_a^U} \right)}{\partial w_j^b} \right) = -\left(\frac{\partial \left(\frac{x_j^{bU}}{x_a^U} \right)}{\partial P} \right), \mathrm{j} = 1,..., B, \tag{3.20}$$

$$\left(\frac{\partial \left(\frac{y^U}{x_a^U} \right)}{\partial F} \right) = -\left(\frac{\partial \left(\frac{1}{x_a^U} \right)}{\partial P} \right) = \left(\frac{\partial x_a^U}{\partial P} \right) \left(\frac{1}{x_a^{U2}} \right), \tag{3.21}$$

$$\left(\frac{\partial \left(\frac{x_j^{bU}}{x_a^U} \right)}{\partial F} \right) = \left(\frac{\partial \left(\frac{1}{x_a^U} \right)}{\partial w_j^b} \right) = -\left(\frac{\partial x_a^U}{\partial w_j^b} \right) \left(\frac{1}{x_a^{U2}} \right), j = 1,..., B, \text{ and} \tag{3.22}$$

$$\left(\frac{\partial \left(\frac{x_i^{bU}}{x_a^U} \right)}{\partial w_j^b} \right) = \left(\frac{\partial \left(\frac{x_j^{bU}}{x_a^U} \right)}{\partial w_i^b} \right), \ \mathrm{i} \neq \mathrm{j},\ \mathrm{i,j} = 1,..., B. \tag{3.23}$$

COROLLARY 1.2: *The unrestricted relative processing demand functions are downward sloping in their own price, i.e.*

$$\left(\frac{\partial \left(\frac{\mathrm{D_j^U}\left(P, \mathbf{w^b}, F\right)}{\mathrm{x_a^U}\left(P, \mathbf{w^b}, F\right)} \right)}{\partial w_j^b} \right) \leq 0,\ \mathrm{j} = 1,..., B. \tag{3.24}$$

PROOF:
The statement made in expression (3.24) is an immediate consequence of the unrestricted NARP function being convex in the price vector (P, $\mathbf{w^b}$). The diagonal entries in the symmetric matrix **[NARP$^{\mathbf{U}}_{\mathbf{ts}}$], t,s** = 1,…,$B$+1, are necessarily positive.

COROLLARY 1.3. *The own-price elasticity of a particular processing input demand function is less than (or equal to) the elasticity of the raw agricultural input with regards to the particular processing input price, i.e.*

$$\mathrm{EL}_{w_j^b}\,\mathrm{D}_\mathrm{j}^\mathrm{U}\left(P,\mathbf{w^b},F\right)\leq \mathrm{EL}_{w_j^b}\,\mathrm{x}_\mathrm{a}^\mathrm{U}\left(P,\mathbf{w^b},F\right),\ \mathrm{j}=1,...,B. \tag{3.25}$$

PROOF:
This is an immediate consequence of corollary 1.2, and the quotient rule.

COROLLARY 1.4: *The unrestricted relative supply function is upward sloping in final output price P, i.e.*

$$\left(\frac{\partial\left(\dfrac{\mathrm{S}^\mathrm{U}\left(P,\mathbf{w^b},F\right)}{\mathrm{x}_\mathrm{a}^\mathrm{U}\left(P,\mathbf{w^b},F\right)}\right)}{\partial P}\right)\geq 0. \tag{3.26}$$

PROOF:
The statement made in expression (3.26) is another immediate consequence of the unrestricted NARP function being convex in the price vector (P, $\mathbf{w^b}$).

COROLLARY 1.5: *The supply function is more (or equally) elastic than the raw agricultural input function with regards to the final output price P, i.e*

$$\mathrm{EL}_P\,\mathrm{S}^\mathrm{U}\left(P,\mathbf{w^b},F\right)\geq \mathrm{EL}_P\,\mathrm{x}_\mathrm{a}^\mathrm{U}\left(P,\mathbf{w^b},F\right). \tag{3.27}$$

PROOF:
This result can be derived directly from corollary 1.4 using the quotient rule.

COROLLARY 1.6: *The unrestricted conversion factor is downward sloping in final output price P, i.e.*

$$\frac{\partial\left(\dfrac{\mathrm{x}_\mathrm{a}^\mathrm{U}(\cdot)}{\mathrm{S}^\mathrm{U}(\cdot)}\right)}{\partial P}=\left(\frac{\mathrm{x}_\mathrm{a}^\mathrm{U}(\cdot)}{\mathrm{S}^\mathrm{U}(\cdot)P}\right)\left(\left(\frac{\partial\,\mathrm{x}_\mathrm{a}^\mathrm{U}(\cdot)}{\partial P}\right)\left(\frac{P}{\mathrm{x}_\mathrm{a}^\mathrm{U}(\cdot)}\right)-\left(\frac{\partial\mathrm{S}^\mathrm{U}(\cdot)}{\partial P}\right)\left(\frac{P}{\mathrm{S}^\mathrm{U}(\cdot)}\right)\right)\leq 0. \tag{3.28}$$

PROOF:
This result is an immediate consequence of corollary 1.5.

COROLLARY 1.7: *The unrestricted raw agricultural demand function is upward sloping in fixed cost F, i.e.*

$$\left(\frac{\partial\,\mathrm{x}_\mathrm{a}^\mathrm{U}\left(P,\mathbf{w^b},F\right)}{\partial F}\right)\geq 0. \tag{3.29}$$

PROOF:
The statement made in expression (3.29) is another immediate consequence of the unrestricted NARP function being convex in the extended vector (P, $\mathbf{w^b}$, F).

THEOREM 2: *The elasticity of the raw agricultural input function* $x_a^U(P, \mathbf{w^b}, F)$ *with regards to final output price P is positive if the elasticity of the NMRP with regards to the final output price P is greater than the share* α_y^U.

PROOF:
In optimum, we have that $NMRP(P, \mathbf{w^b}, x_a^U(P, \mathbf{w^b}, F), K) = NARP^R(P, \mathbf{w^b}, x_a^U(P, \mathbf{w^b}, F), F)$. Differentiating this equality with regards to the final output price P yields:

$$\left(\frac{\partial x_a^U\left(P,\mathbf{w^b},F\right)}{\partial P}\right)=\left(\frac{\left(\frac{\partial NARP^R\left(\cdot\right)}{\partial P}\right)-\left(\frac{\partial NMRP\left(\cdot\right)}{\partial P}\right)}{\left(\frac{\partial NMRP\left(\cdot\right)}{\partial x_0^a}\right)}\right)\Leftrightarrow \tag{3.30}$$

$$EL_P\, x_a^U\left(P,\mathbf{w^b},F\right)=\left(\frac{EL_P\, NMRP\left(\cdot\right)-\alpha_y^U}{-EL_{x_a^C}\, NMRP\left(\cdot\right)}\right),\ \alpha_y^U=\left(\frac{Py}{NARPx_U^a}\right). \tag{3.31}$$

THEOREM 3: When the elasticity of the raw agricultural input function $x_a^U(P, \mathbf{w^b}, F)$ is positive with regards to the final output price P, the supply function is upward sloping in P, i.e.

$$\left(\frac{\partial S^U\left(P,\mathbf{w^b},F\right)}{\partial P}\right)\geq 0. \tag{3.32}$$

PROOF:
This result is an immediate consequence of corollary 1.5.

THEOREM 4: *The elasticity of the raw agricultural input function* $x_a^U(P, \mathbf{w^b}, F)$ *with regards to the processing input price* w_j^b*, j = 1,…, B, is negative if the sum equal to the elasticity of the NMRP with regards to* w_j^b *plus the value share* α_j^U*, is negative.*

PROOF:
In optimum we have that $NMRP(P, \mathbf{w^b}, x_a^U(P, \mathbf{w^b}, F), K) = NARP^R(P, \mathbf{w^b}, x_a^U(P, \mathbf{w^b}, F), F)$. The result in theorem 4 can be derived by differentiating this equality with regards to a particular processing input price w^b_j, j = 1,…, B, which yields

$$\left(\frac{\partial \mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\partial w_j^b}\right)=\left(\frac{\left(\frac{\partial\,\mathrm{NARP^R}\left(\cdot\right)}{\partial w_j^b}\right)-\left(\frac{\partial \mathrm{NM\,RP}\left(\cdot\right)}{\partial w_j^b}\right)}{\left(\frac{\partial \mathrm{NM\,RP}\left(\cdot\right)}{\partial x_0^a}\right)}\right)\Leftrightarrow \tag{3.33}$$

$$EL_{w_j^b}\,\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)=\left(\begin{array}{c} EL_{w_j^b}\,\mathrm{NMRP}\left(P,\mathbf{w^b},\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right),K\right)+\alpha_j^U \\ -EL_{x_a^C}\,\mathrm{NMRP}\left(P,\mathbf{w^b},\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right),K\right)\end{array}\right),$$
$$\alpha_j^U=\left(\frac{w_j^b x_j^b}{NARPx_a^U}\right),\ \mathrm{j}=1,...,B. \tag{3.34}$$

THEOREM 5: *When the elasticity of the raw agricultural input function* $\mathrm{x_a}^{\mathrm{U}}$(*P*, $\mathbf{w^b}$, *F*) *is negative with regards to the processing factor price* w_j^B*, j = 1,…,B, the demand for processing production factor* x_j^B *is downward sloping in its own price, i.e.*

$$\left(\frac{\partial\,\mathrm{D}_{\mathrm{j}}^{\mathrm{U}}\left(P,\mathbf{w^b},F\right)}{\partial w_j^b}\right)\leq 0,\ \mathrm{j}=1,...,\mathrm{B}. \tag{3.35}$$

PROOF:
This result is an immediate consequence of corollary 1.3.

THEOREM 6: *The elasticity of the unrestricted NARP function with regards to the final output price P, equals the value share* α_y^U .

PROOF:
Notice that:

$$\left(\frac{\partial\,\mathrm{NARP^U}\left(\cdot\right)}{\partial P}\right)\left(\frac{P}{\mathrm{NARP^U}\left(\cdot\right)}\right)=\left(\frac{\mathrm{S^U}\left(P,\mathbf{w^b},F\right)P}{\mathrm{x_a^U}\left(P,\mathbf{w^b},F\right)\mathrm{NARP^U}\left(\cdot\right)}\right)=\alpha_y^U. \tag{3.36}$$

THEOREM 7: *The elasticity of the unrestricted NARP function with regards to processing input price* w_j^b*, equals the negative of the value share* α_j^U , j = 1,..., *B*.

PROOF:
Notice that:

$$\left(\frac{\partial \,\mathrm{NARP}^{\mathrm{U}}(\cdot)}{\partial w_j^b}\right)\left(\frac{w_j^b}{\mathrm{NARP}^{\mathrm{U}}(\cdot)}\right) = -\left(\frac{\mathrm{D}_j^{\mathrm{U}}(\cdot)\, w_j^b}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}(\cdot)\,\mathrm{NARP}^{\mathrm{U}}(\cdot)}\right) \tag{3.37}$$
$$= -\alpha_j^U, \ \mathrm{j} = 1,..,B.$$

THEOREM 8: *The elasticity of the unrestricted NARP function with regards to fixed cost F, equals the negative of the value share* α_F^U .

Proof:
Notice that:

$$\left(\frac{\partial \,\mathrm{NARP}^{\mathrm{U}}(\cdot)}{\partial F}\right)\left(\frac{F}{\mathrm{NARP}^{\mathrm{U}}(\cdot)}\right) = -\left(\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(P, \mathbf{w^b}, F\right)\mathrm{NARP}^{\mathrm{U}}(\cdot)}\right) = -\alpha_F^U. \tag{3.38}$$

After having derived eight theorems pertaining to a processing cooperative with a peak coordinator able to restrict members' supplies of the raw agricultural quantity, I now turn my focus towards the adjustment of an unrestricting processing cooperative.

4. The adjustment of an unrestricting processing cooperative

The decisive characteristic of an unrestricting processing cooperative is that the peak coordinator is unable to control members' supplies of the raw agricultural product $\mathrm{x}_{\mathrm{a}}^{\mathrm{C}}$. Without loss of generality I once again divide the optimisation problem into two separate stages. Let the first stage be equal to the primary stage analysed in chapter 3 where the peak coordinator maximises *GRP* for a given raw agricultural quantity x_0^a.

On stage two the peak coordinator of the unrestricting processing cooperative collects and processes the equilibrium raw agricultural quantity identical to the quantity where the NARP function intersects with the inverse of the aggregated raw agricultural supply schedule as depicted in figure 1 above:

$$\mathrm{NARP}\left(P, \mathbf{w^b}, x_0^a, F\right) = \mathrm{MC_a}\left(\mathbf{w^d}, x_0^a; T_a\right). \tag{4.1}$$

Stability requires that the inverse of the aggregated raw agricultural supply function cut the NARP function from below, implying that:

$$\left(\frac{\partial \,\mathrm{MC_a}\left(\mathbf{w^d}, x_0^a; T_a\right)}{\partial x_a^C} - \frac{\partial \,\mathrm{NARP}\left(P, \mathbf{w^b}, x_0^a, F\right)}{\partial x_a^C}\right) = \beta > 0. \tag{4.2}$$

As stated in (Senhaji 2007), the equilibrium raw agricultural quantity x_a^0 is defined implicitly by the equality in expression (4.1) as a function of the output price

P, the processing production factor prices in the vector $\mathbf{w^b}$, the farming input prices in the vector $\mathbf{w^d}$, fixed cost F, and the agricultural policy variable T_a:

$$x_0^a = \mathrm{x}_0^a\left(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a\right). \qquad (4.3)$$

Based on the equality in expression (4.1) and the stability condition in expression (4.2), four theorems related to the equilibrium raw agricultural schedule defined in expression (4.3) are forthcoming:

THEOREM 9: *The equilibrium raw agricultural quantity is upward sloping in the final output price P.*

Proof:
A positive change in the output price P will shift the NARP function depicted in figure 1 upwards, and the equilibrium raw agricultural quantity x_0^a increases:

$$\left(\frac{\partial\,\mathrm{x}_0^a\left(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a\right)}{\partial P}\right) = \left(\frac{\left(\dfrac{\partial \mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial P}\right)}{\beta}\right) = \left(\frac{\left(\dfrac{\mathrm{S}^{\mathrm{R}}(\cdot)}{\mathrm{x}_0^a(\cdot)}\right)}{\beta}\right) \geq 0. \qquad (4.4)$$

THEOREM 10: *The equilibrium raw agricultural quantity is downward sloping in the processing production factor price w_j^b, j=1,..., B.*

Proof:
A positive change in a processing production factor price w_j^b will shift the NARP function depicted in figure 2 downwards, leading to a reduction in the equilibrium raw agricultural quantity x_0^a:

$$\left(\frac{\partial\,\mathrm{x}_0^a(\cdot)}{\partial w_j^b}\right) = \left(\frac{\left(\dfrac{\partial \mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial w_j^b}\right)}{\beta}\right) = -\left(\frac{\left(\dfrac{\mathrm{D}_j^{\mathrm{R}}(\cdot)}{\mathrm{x}_0^a(\cdot)}\right)}{\beta}\right) \leq 0,\ \mathrm{j} = 1,\ldots,B. \qquad (4.5)$$

THEOREM 11: *The equilibrium raw agricultural quantity is downward sloping in fixed cost F.*

Proof:
If the cooperative members decide to raise the collectively determined rent on productive capital K, fixed cost also increases. The NARP function in figure 2 shifts downwards, and the equilibrium raw agricultural quantity x_0^a *decreases:*

$$\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}(\cdot)}{\partial F}\right)=\left(\frac{\left(\frac{\partial \mathrm{NARP}(\cdot)}{\partial F}\right)}{\beta}\right)=-\left(\frac{\left(\frac{1}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)}{\beta}\right)<0. \tag{4.6}$$

THEOREM 12: *If the production factor x_j^d, j=1,…, D, is a normal production factor*[6] *in agriculture, the equilibrium raw agricultural quantity is downward sloping in the input price w_j^d, j=1,…, D.*

Proof:
Let $\mathrm{C_a}(\mathbf{w^d}, x_a^C; T_a)$ denote the aggregated cost function of the cooperative farmers. If the production factor x_j^d, $j = 1,\ldots, D$, is a normal input factor in agriculture, we have that:

$$\left(\frac{\partial \mathrm{S_a^{-1}}\left(\mathbf{w^d}, x_a^C, T_a\right)}{\partial w_j^d}\right)=\left(\frac{\partial \mathrm{MC_a}\left(\mathbf{w^d}, x_a^C; T_a\right)}{\partial w_j^d}\right)>0,\ \mathrm{j}=1,\ldots,D. \tag{4.7}$$

Thus, an increase in the price of a normal production factor x_j^d in agriculture will shift the inverse aggregated raw agricultural supply schedule depicted in figure 1 upwards leading to a reduction in the equilibrium raw agricultural quantity x_0^a:

$$\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}(\cdot)}{\partial w_j^d}\right)=-\left(\frac{\left(\frac{\partial^2 \mathrm{C_a}(\cdot)}{\partial w_j^d \partial x_a^C}\right)}{\beta}\right)=-\left(\frac{\left(\frac{\partial \mathrm{MC_a}(\cdot)}{\partial w_j^d}\right)}{\beta}\right)<0,\ \mathrm{j}=1,\ldots,D. \tag{4.8}$$

$$\mathrm{NARP^E}\left(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a\right)=\mathrm{NARP^R}\left(P, \mathbf{w^b}, \mathrm{x}_0^a(\cdot), F\right). \tag{4.9}$$

Unless the inverse of the aggregated raw agricultural supply schedule intersects with the NARP function at the latter function’s apex, the derivate of the equilibrium NARP function $\mathrm{NARP^E}(P, \mathbf{w^b}, \mathbf{w^d}, F, T_a)$ defined in expression (4.9) with regards to the raw agricultural quantity will not be equal to zero. Accordingly, in the equilibrium pertaining to the unrestricting processing cooperative, we generally have that:

$$\left(\frac{\partial \mathrm{NARP^E}(\cdot)}{\partial P}\right)=\left(\frac{\mathrm{S^R}\left(P, \mathbf{w^b}, \mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)+\left(\frac{\partial \mathrm{NARP^R}(\cdot)}{\partial x_a^C}\right)\left(\frac{\partial \mathrm{x}_0^{\mathrm{a}}(\cdot)}{\partial P}\right)\Leftrightarrow \tag{4.10}$$

[6] The farming production factor x_j^d is a normal input factor when the aggregated marginal cost function of the cooperative farmers denoted by $\mathrm{MC_a}(\mathbf{w^d}, x_a^C)$, is increasing in the input price w_j^d.

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{E}}\left(P,\mathbf{w}^{\mathbf{b}},\mathbf{w}^{\mathbf{d}},F,T_a\right)}{\partial P}\right)\neq\left(\frac{\mathrm{S}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right). \tag{4.11}$$

Likewise, differentiating the equilibrium NARP function with regards to a processing production factor price w_j^b yields:

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{E}}(\cdot)}{\partial w_j^b}\right)=-\left(\frac{\mathrm{D}_{\mathrm{j}}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right)+\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{R}}(\cdot)}{\partial x_a^C}\right)\left(\frac{\partial\,\mathrm{x}_0^{\mathrm{a}}(\cdot)}{\partial w_j^b}\right)\Leftrightarrow \tag{4.12}$$

$$\left(\frac{\partial\,\mathrm{NARP}^{\mathrm{E}}\left(P,\mathbf{w}^{\mathbf{b}},\mathbf{w}^{\mathbf{d}},F,T_a\right)}{\partial w_j^b}\right)\neq-\left(\frac{\mathrm{D}_{\mathrm{j}}^{\mathrm{R}}\left(P,\mathbf{w}^{\mathbf{b}},\mathrm{x}_0^{\mathrm{a}}(\cdot)\right)}{\mathrm{x}_0^{\mathrm{a}}(\cdot)}\right),\ j=1,\ldots,B. \tag{4.13}$$

From expressions (4.12) and (4.13) it is clear that unless the inverse of the aggregated raw agricultural supply schedule is either vertical or intersects with the NARP function at the latter function's maximum, the relative short-run choice functions of the unrestricting processing cooperative cannot be retrieved through the gradient of the equilibrium NARP function in the price vector (P, $\mathbf{w^b}$).

Just like the restricted NARP function defined in expression (3.7) and the unrestricted NARP function defined in expression (3.12), the equilibrium NARP function is nondecreasing in output price P, nonincreasing in processing production factor prices $\mathbf{w^b}$, nonincreasing in fixed cost F, and nonincreasing in the input prices w_j^D, j = 1,…, D, of normal farming production factors. But convexity and positive linear homogeneity in the extended vector (P, $\mathbf{w^b}$, F) does not apply to the equilibrium NARP function. Thus, unambiguous qualitative comparative static results for the relative choice functions of an unrestricting processing cooperative are not forthcoming on the basis of the equilibrium NARP function when the aggregated raw agricultural supply function exhibits a general positive slope due to decreasing returns to scale in agriculture.

5. Conclusion

In this article I have shown that the gradient of the unrestricted NARP function in the price vector (P, $\mathbf{w^b}$) contains the unrestricted relative NARP maximising choice functions of a processing cooperative with a peak coordinator that is able to restrict members' supplies of the raw agricultural product. Unambiguous qualitative comparative static results are derived on the basis of the unrestricted NARP function proven to be convex in the extended vector (P, $\mathbf{w^b}$, F). The large amount of economic information embedded in the curvature properties of the unrestricted NARP function has not been stated explicitly, and resembles the curvature properties of the profit per worker function related to Labour-Managed firm analysed exhaustively by (Neary 1988), (Kahana 1989), and (Wolfstetter 1992).

When the aggregated raw agricultural supply schedule is not completely vertical, but instead exhibits a general positive slope, unambiguous qualitative comparative static results for the relative choice functions of an unrestricting processing cooperative that resemble processing cooperatives with an open-membership policy, are not forthcoming on the basis of the NARP function. Most European agribusiness processing cooperatives are unrestricting in the sense that the daily management cannot control members' supplies of the raw agricultural product. Therefore, the lack of unambiguous qualitative comparative static results for the relative choice functions of unrestricting processing cooperatives poses a great challenge.

Principal-agent conflicts are embedded in the internal relationships between the collective principal and the agent represented by the peak coordinator being responsible for the daily management. Bargaining situations arise between different heterogeneous groups of farmers within the cooperative. (LeVay 1983) points out the need for analysis where the peak coordinator does not operate with only one single objective. The multi-criteria approach may prove to be an appropriate tool in analysis of cooperative adjustments where the peak coordinator is forced by the collective principal to consider numerous objectives in the daily management of the society's common business.

(LeVay 1983) and (Sexton 1984) describe other plausible objectives that the daily management of a processing cooperative may pursue on behalf of the members. Qualitative comparative static results similar to those derived for the restricting processing cooperative are also derivable for the adjustments corresponding to the short-run maximisation of the patronage dividend and members' total profits.

There is a close resemblance between the plausible adjustments of processing cooperatives on the one hand, and supply- and purchasing cooperatives on the other. Unambiguous qualitative comparative static results for plausible adjustments pertaining to supply- and purchasing cooperatives can therefore be derived on the basis of the average purchasing cost function.

Finally, qualitative long-run comparative static results for processing and supplying cooperatives must be derived. In the long run, even productive capital K with the price w_K, will be endogenous. If the peak coordinator maximises the long-run average revenue product, the indirect objective function ARP(P, $\mathbf{w^b}$, w_K) is a support function of the implicit relative production possibilities set $\tilde{T}$ defined as:

$$\tilde{T} = \left\{ \begin{array}{l} \left(\frac{y}{x_a^C} \; \frac{\mathbf{x^b}}{x_a^C} \; \frac{K}{x_a^C} \right) : \left(P\left(\frac{y}{x_a^C} \right) - \left(\sum_{j=1}^{B} w_j^b \left(\frac{x_j^b}{x_a^C} \right) \right) - w_K K \right) \leq \\ \mathrm{ARP}\left(P, \mathbf{w^b}, w_K \right) \text{for all } \mathbf{w^b} > \mathbf{0_B} \text{ and } w_K > 0. \end{array} \right\} \quad (5.1)$$

Unambiguous long-run qualitative comparative static results for an ARP-maximising processing cooperative are forthcoming on the basis of the support function ARP(P, $\mathbf{w^b}$, w_K). Similar long-run comparative static results can be derived for processing cooperatives pursuing one of the other plausible objectives described by, among others, (LeVay 1983) and (Sexton 1989).

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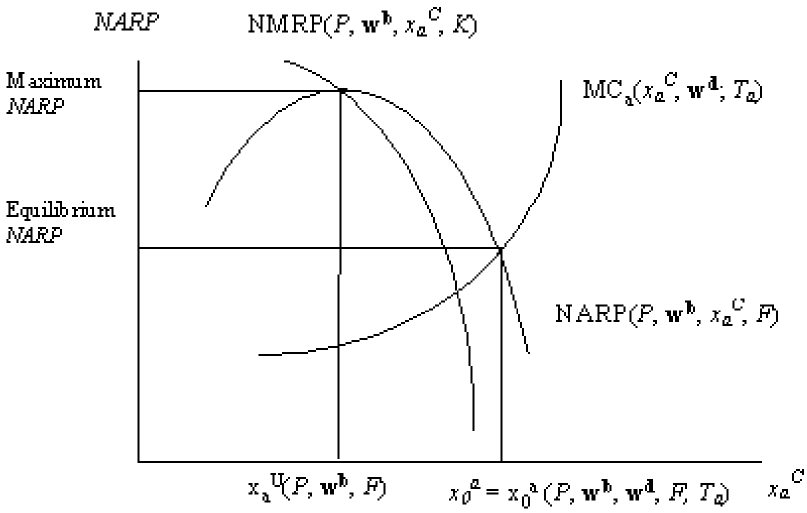

Figure 1
A restricting processing cooperative secures members the maximum *NARP* by collecting and processing the raw agricultural quantity $x_a^U(P, \mathbf{w^b}, F)$. When the processing cooperative is unrestricting, the adjustment will be characterised by the equilibrium *NARP* together with the equilibrium raw agricultural quantity x_0^a.

Appendix 1: Proofs of statements (*III*)

The first statement III.1 reflects how the restricting processing cooperative contributes its maximum effort in order to meet fixed cost. The peak coordinator takes the cooperative firm out of business once the variable processing costs cannot be met so that *ARP* turns negative.

Statement III.2 says that the unrestricted NARP function is nondecreasing in the final output price *P*. When $P_2 \geq P_1$, we have that

$$\mathrm{NARP^U}\left(P_2,\mathbf{w^b},F\right)=$$

$$\left(P_2\left(\frac{\mathrm{S^U}\left(P_2,\mathbf{w^b},F\right)}{\mathrm{x_a^U}\left(P_2,\mathbf{w^b},F\right)}\right)-\sum_{j=1}^{B}w_j^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P_2,\mathbf{w^b},F\right)}{\mathrm{x_a^U}\left(P_2,\mathbf{w^b},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P_2,\mathbf{w^b},F\right)}\right)\geq$$

$$\left(P_2\left(\frac{\mathrm{S^U}\left(P_1,\mathbf{w^b},F\right)}{\mathrm{x_a^U}\left(P_1,\mathbf{w^b},F\right)}\right)-\sum_{j=1}^{B}w_j^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P_1,\mathbf{w^b},F\right)}{\mathrm{x_a^U}\left(P_1,\mathbf{w^b},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P_1,\mathbf{w^b},F\right)}\right)\geq \quad (1)$$

$$\left(P_1\left(\frac{\mathrm{S^U}\left(P_1,\mathbf{w^b},F\right)}{\mathrm{x_a^U}\left(P_1,\mathbf{w^b},F\right)}\right)-\sum_{j=1}^{B}w_j^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P_1,\mathbf{w^b}\right)}{\mathrm{x_a^U}\left(P_1,\mathbf{w^b},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P_1,\mathbf{w^b},F\right)}\right)=$$

$$\mathrm{NARP^U}\left(P_1,\mathbf{w^b},F\right)\Leftrightarrow$$

$$\mathrm{NARP^U}\left(P_2,\mathbf{w^b},F\right)\geq\mathrm{NARP^U}\left(P_1,\mathbf{w^b},F\right)\Leftrightarrow \quad (2)$$

$$\mathrm{NARP^U}\left(P_2,\mathbf{w^b},F\right)-\mathrm{NARP^U}\left(P_1,\mathbf{w^b},F\right)=\Delta NARP^U \geq 0. \quad (3)$$

Statement III.3 says that the unrestricted NARP function is nonincreasing in any of the *B* processing input prices. When $\mathbf{w_2}^\mathbf{b} \geq \mathbf{w_1}^\mathbf{b}$, we have that

$$\mathrm{NARP^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)=$$

$$\left(P\left(\frac{\mathrm{S^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}\right)-\sum_{j=1}^{B}w_{2j}^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}\right)\leq$$

$$\left(P\left(\frac{\mathrm{S^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}\right)-\sum_{j=1}^{B}w_{1j}^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)}\right)\leq \quad (4)$$

$$\left(P\left(\frac{\mathrm{S^U}\left(P,\mathbf{w}_1^\mathbf{b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_1^\mathbf{b},F\right)}\right)-\sum_{j=1}^{B}w_{1j}^b\left(\frac{\mathrm{D}_j^{\mathrm{U}}\left(P,\mathbf{w}_1^\mathbf{b},F\right)}{\mathrm{x_a^U}\left(P,\mathbf{w}_1^\mathbf{b},F\right)}\right)-\frac{F}{\mathrm{x_a^U}\left(P,\mathbf{w}_1^\mathbf{b},F\right)}\right)=$$

$$\mathrm{NARP^U}\left(P,\mathbf{w}_1^\mathbf{b},F\right)\Leftrightarrow$$

$$\mathrm{NARP^U}\left(P,\mathbf{w}_2^\mathbf{b},F\right)\leq\mathrm{NARP^U}\left(P,\mathbf{w}_1^\mathbf{b},F\right)\Leftrightarrow \quad (5)$$

$$\mathrm{NARP}^{\mathrm{U}}\left(P,\mathbf{w}_2^{\mathbf{b}},F\right)-\mathrm{NARP}^{\mathrm{U}}\left(P,\mathbf{w}_1^{\mathbf{b}},F\right)=\Delta\,\mathrm{NARP}^{\mathrm{U}}\leq 0. \tag{6}$$

Statement II.A.5 says that the unrestricted NARP function is positively linearly homogeneous in the extended vector (P, $\mathbf{w}^{\mathbf{b}}$, F). Let the superscript «T» denote the transpose operator. Notice that

$$\mathrm{NARP}^{\mathrm{U}}\left(tP,t\mathbf{w}^{\mathbf{b}},tF\right)=\max_{y,\mathbf{x}^{\mathbf{b}},x_a^C}\left\{\frac{tPy-t\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x}^{\mathbf{b}}-tF}{x_a^C}\mid y\leq \mathrm{f}\left(\mathbf{x}^{\mathbf{b}},x_a^C\right)\right\}=$$

$$t\max_{y,\mathbf{x}^{\mathbf{b}},x_a^C}\left\{\frac{Py-\mathbf{w}^{\mathbf{b}\mathrm{T}}\mathbf{x}^{\mathbf{b}}-F}{x_a^C}\mid y\leq \mathrm{f}\left(\mathbf{x}^{\mathbf{b}},x_a^C\right)\right\}=t\,\mathrm{NARP}^{\mathrm{U}}\left(P,\mathbf{w}^{\mathbf{b}},F\right). \tag{7}$$

In order to prove statement III.5 saying that the unrestricted NARP function is convex in (P, $\mathbf{w}^{\mathbf{b}}$), we first introduce three different price vectors $\mathbf{P_1}$, $\mathbf{P_2}$, and $\mathbf{P_3}$ containing (B+1) strictly positive final output- and processing input factor prices. The three corresponding optimal vectors of relative supplies equal to [$S^U(\mathbf{P_i}, F)/x_a^U(\mathbf{P_i}, F)$] and relative processing input demands [$D_j^U(\mathbf{P_i}, F) / x_a^U(\mathbf{P_i}, F)$] with negative signs, j = 1,…,B, and i = 1,…,3, are given by the relative quantity vectors $\mathbf{y_1^*}$, $\mathbf{y_2^*}$, and $\mathbf{y_3^*}$. Let α ε (0,1), and define $\mathbf{P_3}$ as the weighted average of the former two price vectors: $\mathbf{P_3} = \alpha\,\mathbf{P_1} + (1-\alpha)\,\mathbf{P_2}$. Again the superscript «T» denotes the transpose operator. Based on these definitions, I prove that $\mathrm{NARP}^{\mathrm{U}}(P, \mathbf{w}^{\mathbf{b}}, F)$ is convex in (P, $\mathbf{w}^{\mathbf{b}}$) by stating first of all that

$$\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P_1},F\right)=\left(\mathbf{P}_1^{\mathrm{T}}\mathbf{y}_1^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_1},F\right)}\right)\geq\left(\mathbf{P}_1^{\mathrm{T}}\mathbf{y}_3^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P}_3,F\right)}\right),\text{ and} \tag{8}$$

$$\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P_2},F\right)=\left(\mathbf{P}_2^{\mathrm{T}}\mathbf{y}_2^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P}_2,F\right)}\right)\geq\left(\mathbf{P}_2^{\mathrm{T}}\mathbf{y}_3^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P}_3,F\right)}\right). \tag{9}$$

Multiply both sides of inequalities (8) and (9) by α and (1- α) respectively, and add the two appearing inequalities in order to get

$$\begin{aligned}&\alpha\,\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_1,F\right)+\left(1-\alpha\right)\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_2,F\right)\geq\\&\left(\alpha\left(\mathbf{P}_1^{\mathrm{T}}\mathbf{y_3^*}\right)+\left(1-\alpha\right)\left(\mathbf{P}_2^{\mathrm{T}}\mathbf{y}_3^*\right)-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_3},F\right)}\right)=\\&\left(\left(\alpha\mathbf{P}_1^{\mathrm{T}}+\left(1-\alpha\right)\mathbf{P}_2^{\mathrm{T}}\right)\mathbf{y_3^*}-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_3},F\right)}\right)=\\&\left(\mathbf{P}_3^{\mathrm{T}}\mathbf{y}_3^*-\frac{F}{\mathrm{x}_{\mathrm{a}}^{\mathrm{U}}\left(\mathbf{P_3},F\right)}\right)\equiv\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P_3},F\right)\Leftrightarrow\end{aligned} \tag{10}$$

$$\alpha \, \mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_1, F\right) + \left(1-\alpha\right)\mathrm{NARP}^{\mathrm{U}}\left(\mathbf{P}_2, F\right)$$
$$\geq \mathrm{NARP}^{\mathrm{U}}\left(\alpha \mathbf{P}_1 + \left(1-\alpha\right)\mathbf{P}_2, F\right) \tag{11}$$

From expression (11) we see that the unrestricted NARP function is convex in the price vector (P, $\mathbf{w}^{\mathbf{b}}$). A similar procedure can be undertaken in order to prove that the unrestricted NARP function is convex in the extended vector (P, $\mathbf{w}^{\mathbf{b}}$, F).

Finally, statement III.6 follows directly from the Viner-Wong envelope theorem.