Networks as a Model of Governance: The case of Credit Mutuals

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Abstract

This paper presents a model of network formation for credit mutuals (CM). The importance of the subject resides in the fact that CM as well as many other types of mutual organizations tend often to integrate into network-like structures. Sometimes these structures are so imposing that they have become, when taken as a network, the primary, or one of the primary, financial institutions in the country. However, little is known about the factors that influence their formation and stability. We present the model in three steps. First we develop a simple model of a CM that maximizes member monetary and non-monetary surplus. Then we define the basic structure of a network that separates it from a conglomerate with subsidiaries, and, based on that definition, build a model of a network by assuming absence of default. This basic model allows us to draw a number of predictions about the factors that influence the formation of network. Amongst the more interesting ones we may cite that operational efficiency is not the (only) reason for networks to be desirable. Other factors, such as diversifying the mixture of membership (savers vs. borrower members) may also have an influence. Further, members may draw benefits from networking from benefiting from additional monetary and non-monetary surpluses. In the
third step, we incorporate default risk to the model and derive a new set of equilibrium conditions. These conditions suggest that increases in (expected) default risk will discourage network formation and that, in exiting networks, CM with high loan default rates will tend to be marginalized from the decision process.

1 Introduction

In this paper we model inter-firm relationships in the form of networks of credit mutuals (CM). Generally speaking, we define a network as a private collective (of enterprises) deriving mutual benefits from pooling common (or network) resources, by establishing an organization with a representation governance to procure and manage the designated segments of common resources. Networks exist in a number of industries but our focus is on those created by CM. Our interest focuses on factors that affect incentives to the formation and stability of these networks.

Although networks are a very modern and striving area of research in organization theory, CM are, in fact, pioneers in organizing networks to exploit economies of scale and scope and control uncertainty. The concept was deployed by CM in Europe as early as the XIX century. Some of the systems of CM created then are today, one-hundred years later, when taken as a network the, or one of the, primary financial institution of the country, exceeding in size sometimes by a multiple the largest joint stock institutions. This is the case in several European countries including Austria, Finland, France, Germany, and Netherlands. In the case of France and Germany CM control about 50% of financial assets in the economy.\textsuperscript{1} While the economics of networking is still a hot subject of academic debate, researchers have identified networks to be useful in situations where:

- Mergers may be too risky in view of the uncertainties of the project. In these cases networking allows to engage in reversible commitments (in this sense networks create valuable real options). Ex. Joint mining or technology development projects.

- Exploitation of economies of scale and scope in the production of inputs while conserving independence of the contracting parties (protection of residual property rights) is desired. Ex. Printing, banking, biotechnology.

- Providers of complementary services join in a long-term relationship to provide customers expanded value through coordination in the provision of the services. Ex. Passenger airline industry, computer software.

\textsuperscript{1}This is, incidentally, also the case of Quebec, where desjardins controls about 45% of total provincial financial assets and serves to about 75% of the population.
The second application is the one pioneered by CM with success measured by the diffusion at international level and the enormous scale and complexity of some of the existing networks of CM. Whenever networks (or joint ventures/alliances) are created, governance structures become a necessity. These governance structures insure that the alliances achieve their goal and at the same time that the contracting parties adhere to engagements and to fair play. In networks all participating members subscribe to a contract that establishes the rights and obligations of each and all members. The institutions that result from these agreements are structures that have complex rules of operations designed to: i) Generate services for member CM including pooling of inputs, joint management of infrastructure and pooling of risk across the system; ii) Insure a strategic coordination and leadership of the network; iii) Enforce compliance of engagements by all parties under the subscription agreement; iv) Provide protection of member-shareholders residual property and control rights.

Networks of CM have shown a remarkable resiliency and adaptability to disturbances. Most systems are in place for almost a century and show no sign of weakening. While CM are sometimes described as an "appropriate technology for backward economies" [2] they appear to have either hold their place or advanced against other financial intermediaries with different governance structure—particularly joint-stock banks. The era of de-regulation started in the 70’s has provided joint-stock banks new opportunities to expand on products and services while the increasing competition reduced the risk of opportunism by lenders in arms-length relationships. These developments have weakened the arguments that support the creation of mutual financial intermediaries in the first place. Networks have responded by adapting their governance structure and internal distribution of responsibilities to the new environment. Further, with minor institutional variations they have been implemented in cultural, legal and economic environments as varied as those found in Finland, Germany, Australia, Japan, Quebec, Brazil and Korea to mention a few.

Despite the widespread presence of networks of CM worldwide, there is almost no literature about their functioning, and even less about the economic dynamics that rule these inter-firm relationships. In this paper we focus on networks of depository type of CM, however, structures of similar characteristics and complexity exist among mutuals in other industries such as insurance cooperatives (e.g. Promutuel in Quebec); health management (e.g. Statutory Health Insurance mutuals in Germany and the Mutualité Française, both the major health insurers in the countries); community banks (Cajas in Spain and Sparkassen in Germany, respectively the largest institution, taken as networks, in their respective countries) to mention just a few. Thus, the interest of understanding these structures thus transcends vastly the field of cooperative depository institutions.

Networks have been subject of research using other approaches. Specifically, networks are an hybrid form of governance of transactions in the theory of transaction cost economics (TCE)[7]. Modeling of the CM based on TCE principles is available [1]. This paper is an attempt to formalize the modeling of this particular form
of hybrid using different tools. In this model, although not strictly based on TCE principles, there are two basic ingredients of a TCE based analysis: i) there is a transaction or contractual relation that is established between parties (the CM) that gives rise to contractual hazards; ii) parties to the contract choose among governance mechanisms –i.e. alliance or merger– to govern the relation. What has not (yet) been included in the model is the differential costs associated to the governance choice. We keep however one focus common to TCE, the governance choices decision makers face when confronting contractual hazard—one of the three “critical dimensions” of transactions in the TCE framework—to insure that property rights are protected. In this sense the model has elements of the property rights approach to institutional economics.

The organization of the paper is as follows. In section 2 we first present the basic model of a CM, one in which no default risk on loans exist. We then tie individual CM into a network using basic assumptions that distinguish these from a normal subsidiary and develop a model of the network. The benefits of the network become evident by comparing surpluses to members as stand alone units and as members of a network. In section 3 we introduce a non-random loan default rate and evaluate the impact this has on the surplus generated to members when becoming member of a network. In both these sections we draw predictions about the behavior of CM with respect to other CM and the network. In section 4 we draw conclusions.

2 The Basic Model

2.1 Individual CM

Assume an individual CM that serves a population of \( N \) agent-members. This population of members can be described as follows:

**Definition 1** Borrowing members: Each borrowing member \( b = 1, \ldots, N_b \) displays a utility \( U_b(.) \) that results from the loan obtained from the CM:

\[
U_b(r_L) = \begin{cases} 
(r_b - r_L)L_b & \text{if } r_L < r_b \\
0 & \text{otherwise}
\end{cases}
\]

with
- \( L_b \): the principal of the loan engaged by \( b \),
- \( r_L \): the interest rate applied by the CM on the loan,
- \( r_b \): the best alternative loan rate obtainable in the market for a loan of similar characteristics available to member \( b \).
That is, the borrowing member’s utility will be non-zero in case that the CM charges loan rates that are competitive with respect to alternative sources. Assume that the distribution $\{\tau_b\}_{b=1}^{N_b}$ of the population of $N_b$ borrowing members is uniform:

$$\tau_b \sim P_b(\cdot) : \text{Uniform } [r_b^-, r_b^+]$$

We define $m_b = r_b^+ - r_b^-$, a measure of the dispersion of rates charged on loans.

**Definition 2** Saving members: Each saving member $s = 1, ..., N_s$ displays a utility $U_s(\cdot)$ resulting from the savings operation completed with the CM:

$$U_s(r_D) = \begin{cases} (r_D - \tau_s)D_s & \text{if } r_D > \tau_s \\ 0 & \text{otherwise} \end{cases}$$

with

- $D_s$: the amount of the deposit made by $s$,
- $r_D$: the interest rate obtained from the CM on the deposit,
- $\tau_s$: the best alternative deposit rate obtainable in the market for a deposit of similar characteristics available to member $s$.

The same qualification made with respect to rates on loans applies to rates on deposits. The difference is that in this case it applies to deposit rates. Assume that the distribution of the population of saving members is, like that of borrowing members, uniform:

$$\tau_s \sim P_s(\cdot) : \text{Uniform } [r_s^-, r_s^+]$$

We define $m_s = r_s^+ - r_s^-$, a measure of the dispersion of rates paid on deposits. Based on these distribution we can define the following:

**Definition 3** The fund offer and demand functions:

1) Funds demand function (loan demand function):

$$L(r_L) = \Pr [r_L < \tau_b] \times \sum_{b=1}^{N_b} L_b = N_b \left[ \frac{r_b^+ - r_L}{m_b} \right]$$

2) Funds offer function (deposit offer function):

$$D(r_D) = \Pr [r_D > \tau_s] \times \sum_{s=1}^{N_s} D_s = N_s \left[ \frac{r_D - r_s^-}{m_s} \right]$$

**Remark 4**

1) $N = N_b + N_s$

2) Without loss of generality, assume that: $L_b = 1\forall b$ and $D_s = 1\forall s$: *(Assumption 1)*
To insure that sources and uses of funds are balanced we need a balance sheet restriction. Thus,

**Definition 5** Budget (or balance sheet) equation:

\[ L(r_L) = D(r_D) + G \]

with \( G > 0 \) ⇒ **External debt vs.** \( G < 0 \) ⇒ **Investments other than loans.**

With this elements in place we can now define the dynamics of the functioning of the CM. We first start with a profit function for the CM. Note that the assumption is not that the CM is somehow maximizing profits. On the contrary, as is standard in modeling CM, the benefits for members come from different sources.

**Definition 6** The CM profit (or exedent) is:

\[ \Pi(r_L, r_D) = (1 + r_L)L(r_L) - [(+r_D)D(r_D) + (1 + r_G)G + C] \]

With:

- \( r_G \): interest rate received if \( G < 0 \) or paid if \( G > 0 \)
- \( C \): total cost of operations : \( C = c_L L(r_L) + c_D D(r_D) \) where \( c_L (c_D) \) is the marginal cost of management of loan (deposit) accounts.

**Definition 7** Non-monetary surplus of members:

1) Aggregated non-monetary surplus of borrowing members:

\[ SL(r_L) = \int_{r_L^-}^{r_L^+} U_b(r_L)dP_b(\varphi_b) = N_b(r_b^+ - r_L) - \frac{1}{2}N_b m_b \]

2) Aggregated non-monetary surplus of saving members:

\[ SL(r_D) = \int_{r_D^-}^{r_D^+} U_s(r_D)dP_s(\varphi_s) = N_s(r_D^+ - r_s) + \frac{1}{2}N_s m_s = N_s(r_D - r_s) - \frac{1}{2}N_s m_s \]

That is, the non-monetary surplus measures the difference between the rate the borrower is willing to par (earn) for a loan (on a deposit) and the loan rate offered by the CM for the same. This is richer than the definition usually given of surplus as a difference between observed rates and rates charged (paid) on the loan (deposit) incorporating the possibility that no credit (deposit) may be feasible elsewhere.
Definition 8 Objective function of the CM:

A reasonable objective function for the CM is to maximize the aggregated surplus of members. This is composed of the monetary surplus originated in transactions that they execute with the CM and of the monetary surplus realized from the distribution of operating earnings (dividends) generated by the institution.

\[
\max_{(r_L, r_D)} \quad F(r_L, r_D) = \theta_L SL(r_L) + \theta_D SD(r_D) + \Pi(r_L, r_D)
\]

subject to

\[
L(r_L) = D(r_D) + G
\]

Where \(\theta_h (h = L \text{ (borrowers)}, D \text{ (savers)})\) indicates the weight allocated to the group \(h\) in the allocation of the total non monetary surplus. When \(\theta_D < \theta_L (\theta_D > \theta_L)\) we say that the CM is borrower (saver) dominated.\(^2\) This formulation of the maximization problem of a mutual is consistent with the existing literature [e.g., [5], [6]]. Knowing that \(\theta_D + \theta_L = 1, \theta_h \geq 0\). Providing the one-man/one-vote rule, we can argue that:

\[
\theta_D = \frac{N_s}{N}, \theta_L = \frac{N_b}{N}.
\]

i.e. that the borrower (saver) “bias” is a function of the proportion of savers and borrowers in the CM.

The FOC of the maximization problem yields the following equilibrium rates

\[
\begin{align*}
    r^*_L &= \frac{r_G + c_L - \theta_L m_b}{2} \\
    r^*_D &= \frac{r_G - c_D + \theta_D m_s}{2}
\end{align*}
\]

The index \(i\) means that it is the optimal allocation of an individual CM. That is, the equilibrium rate is a function of the rate, \(r_G\), paid to (earned from) external (funding investments), the cost of operating loan and deposit accounts and the weight of the group \(h\) in the allocation of the total non monetary surplus. It is easy to see that:

\[
\begin{align*}
    r^*_L &\downarrow \text{ if: } r_G, c_L, \theta_L \downarrow \\
    r^*_D &\uparrow \text{ if: } r_G, c_D, \theta_D \downarrow
\end{align*}
\]

\(^2\)The borrower (saver) domination in CM is a point of considerable debate in the literature about this type of institutions. Hart and Moore [3], making use of the median Voter Theorem of [4] show that in CM-like institutions, even if it is (net) borrower dominated, the generation of surpluses that may be paid out in the form of dividends or other services will restrain it from offering lending rates at "firesale" levels. The Median Voter Theorem predicts that the outcome will be that which represents the median member and not that of either extreme. Moreover, if decisions are such that members in one or the other extreme of the preference scale desert, a shift in the median will occur with the old median members shifting in direction of the deserters and a new median appearing among members at the opposite side of the deserters giving more weight to their preferences in future decisions. In this paper we use a simple function in which the level of bias depends upon the proportion of members as net borrower (savers). The deserting effect will keep the bias within tolerable levels for both types of members.
When the representation of borrower members increases (borrower bias) loan rates go down. Equally when representation of savers (saver bias) goes up, deposit rates go up.

3 Collectives

When firms (CM or otherwise) decide to create collectives they can choose to merge thus pooling property rights, or establish a long term hybrid relation in which the members of the collective keep legal individuality but pool consequential portions of their operations. We will analyze the decision process in two steps. In the first the CM decide whether they will form a collective. This implies analyzing the benefits of forming a collective whether it takes the form of a network or a merger. In the second step, the analysis focuses on these two choices. We continue assuming assuming default free assets. First we define a system of CM organized in a collective. Such a system implies three attributes:

1. A pooling of operations
2. A common allocation of resources
3. An allocation of property rights. The fundamental difference between the two collectives is that in a network the individual CM maintain legal identity, while in a merger this identity is lost. This difference has implications in the distribution of residual rights that will influence the decision.

For simplicity we consider a collective system consisting of two CM: 1, 2. The pooling of operations that we assume here is complete. This implies a unique pair of marginal costs supported by the two CM. In practice pooling is partial, largely as a function of whether an input is poolable or not. For example, buildings are usually not poolable, but ATM and computer services are. The common allocation of resources implies the creation of an internal capital market.

First we need to determine the collective-wide funds demand and offer functions. We assume here that the two populations of members, \( \{ b \}_{b=1}^{N_b}, \{ s \}_{s=1}^{N_s} \) and \( \{ b \}_{b=1}^{N_2}, \{ s \}_{s=1}^{N_2} \), in the two CM, 1 and 2, possess homogeneous preferences. A direct result of this assumption is that the distributions of \( \bar{r}_b \) and \( \bar{r}_s \) in 1 and 2 are the same. The aggregated offer and demand functions are thus:

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3In organization theory jargon this would rather be called an "alliance." We keep the expression network since most collectives of CM are constituted of tens, or hundreds or even thousands of members, although for modeling purposes we limit ourselves to two members.
\[ L(r_L) = (N_b^1 + N_b^2) \left[ \frac{r_b^+ - r_L}{m_b} \right] = N_b^n \left[ \frac{r_b^+ - r_L}{m_b} \right] \]

\[ D(r_D) = (N_s^1 + N_s^2) \left[ \frac{r_D - r_s^-}{m_s} \right] = N_s^n \left[ \frac{r_D - r_s^-}{m_s} \right] \]

with index n to designate the collective, and:

\[ N_b^n = (N_b^1 + N_b^2); N_s^n = (N_s^1 + N_s^2) \]

Let us next establish the profits from operations and sharing rules involved in a collective. We suppose default-free loans. Therefore, the profit attributable to the CM \( i \) \((i = 1, 2)\) is as follows:

\[
\Pi_i^n(r_L, r_D) = \left( \frac{N_b^i}{N_b^n} \right) (1 + r_L - c_L^n)L^n(r_L) - \left( \frac{N_s^i}{N_s^n} \right) (+r_D + c_D^n)D(r_D) + \left( \frac{N_i}{N^n} \right) (1 + r_G)G^n + C
\]

where:

- \( G^n = L^n(r_L) + D(r_D) \), the collective budget equation (and the relation that supports the internal capital market)
- \( C^n = c_L^n L^n(r_L) + c_D^n D(r_D) \), the total operating cost of the collective
- \((c_L^n, c_D^n)\), the pair of marginal costs following the pooling of operations in the collective.\(^4\)

We note that the total operations costs and the capitalized budget adjustment flow \((1 + r_G)G^n\) are shared as function of the transaction attributed to each CM in the collective.

**Definition 9** The collective objective function:

As in the case of individual CM reasonable objective function for the collective is to maximize the aggregated surplus of members. This is composed of the monetary surplus originated in transactions that they execute with the collective and of the

\(^4\)The pair \((c_L^n, c_D^n)\) is a complex function. Economies of scale arguments—one important reason why CM pool resources—would suggest that, *ceteris paribus*, it is likely to be lower than for the individual CM. Transaction cost economics arguments, however, suggest that: i) governance costs may increase in a network as bureaucratic structures required to govern the relationship are setup; ii) adaptation costs to market disturbances—including default rates on loans—that lead to haggling between the parties would decrease with the complexity of the contractual relationship. An explicit consideration of the different governance factors that influence the pair \((c_L^n, c_D^n)\) is beyond the scope intended for the paper.
monetary surplus realized from the distribution of operating earnings (dividends) it generates.

\[
\max_{\{r_L, r_D\}} F(r_L, r_D) = [\theta^n_L S^n_L(r_L) + \theta^n_D S^n_D(r_D)] + [\gamma_1 \Pi^n_1(r_L, r_D) + \gamma_2 \Pi^n_2(r_L, r_D)]
\]

subject to \(L^n(r_L) = D^n(r_D) + G^n\)

with

\[
S^n_L(r_L) = N^n_b (r_L^b - r_L) - \frac{1}{2} N^n_b m_b
\]

\[
S^n_D(r_D) = N^n_s (r_D^s - r_s) - \frac{1}{2} N^n_s m_s
\]

and

- \(\theta^n_D + \theta^n_L = 1, \theta^n_h \geq 0\), and given the one-man/one-vote rule, as in the individual CM:

\[
\theta^n_D = \frac{N^n_s}{N^n}, \theta^n_L = \frac{N^n_b}{N^n}.
\]

- \(0 < \gamma_i < 1\), and \(\gamma_1 + \gamma_2 = 1\), where \(\gamma_i\) indicates the bargaining power of CM \(i\) in the collective organization. One obvious way to define \(\gamma_i\) is to make it a function of the relative size of the CM that enter into the collective.

This new objective function allows us to obtain collective equilibrium conditions. Clearing rates is one of them. The FOC yields the following optimum rates solution on the collective of CM:

\[
r^n_{L*} = \frac{1}{2 \sum_i \gamma_i} \left[ \sum_i \gamma_i \left( \frac{N^n_i}{N^n_b} (r_L^b - 1 + c^n_L) + \frac{N^n_i}{N^n} (1 + r_G) \right) - \theta^n_L m_b \right]
\]

\[
r^n_{D*} = \frac{1}{2 \sum_i \gamma_i} \left[ \sum_i \gamma_i \left( \frac{N^n_i}{N^n_s} (r_D^s - 1 + c^n_D) + \frac{N^n_i}{N^n} (1 + r_G) \right) - \theta^n_D m_s \right]
\]

The interpretation is not much different from that we obtained for individual CM rates and it is still valid that

\[
r^n_{L*} \downarrow \text{ if: } r_G \downarrow, c^n_L \downarrow, \theta^n_L \uparrow
\]

\[
r^n_{D*} \uparrow \text{ if: } r_G \uparrow, c^n_D \downarrow, \theta^n_D \uparrow .
\]

Of more interest is that, by comparing the optimum obtained for individual CMs and CMs in a collective we observe

\[
r^n_{L*} < r^n_L \text{ if: } \frac{\theta^n_L}{\theta^n_L} < \sum_i \gamma_i \frac{N^n_i}{N^n_b} \text{ and/or } c^n_L < c^L
\]

\[
r^n_{D*} > r^n_D \text{ if: } \frac{\theta^n_D}{\theta^n_D} > \sum_i \gamma_i \frac{N^n_i}{N^n_s} \text{ and/or } c^n_D < c^D
\]
Other the obvious economy in operations costs ($c^o_L < c^i_L$ and $c^o_D < c^i_D$), CM members CM members in collectives may benefit from advantageous rates from the gain of control share the collective organization can generate. For example, borrowers (savers)-dominated CMs will generate more attractive loan (deposits) rates once organized in a collective system that is dominated by borrowers (savers). In contrast, collectives of CMs with different orientations (borrowers-dominated CM with savers-dominated CM) will generate a sort of dilution of control and then less competitive rates over both the two sides (i.e., loans and deposits).

**Result 1:** Operational efficiency is not the only rationale for collectives. Keeping operational efficiency’ effect null, the differences in the mixture of members’ types in the collective and the individual CM can also make collectives worthwhile.

## 4 Economic Value of Collectives

### 4.1 High-value Collectives

We can assess the value of a pooling arrangement for members by looking at the welfare effect of the same. To do this we start by defining the total welfare measure. For an individual CM environment, the total welfare may be given by:

$$SL(r^*_L) + SD(r^*_D) + \Pi(r^*_L, r^*_D)$$

To compare welfare outputs in both individual CMs and Collective of CMs, we have respectively

$$TW_{\text{Individual}} = \sum_i [SL^i(r^*_L) + SD^i(r^*_D) + \Pi^i(r^*_L, r^*_D)]$$

$$TW_{\text{Collective}} = [SL^n(r^{**}_L) + SD^n(r^{**}_D)] + \sum_i \Pi^n_i(r^{**}_L, r^{**}_D)$$

with which we can evaluate the welfare improvement of collective formation. To evaluate the welfare improvement the formation of collective generates, we use the total welfare’s increment:

$$\Delta TW = TW_{\text{Collective}} - TW_{\text{Individual}}$$

**Result 2:** Whatever the values assigned to ($\gamma_1, \gamma_2$), and even if we assume that the collective produces no operational cost gains (individual vs. collective), i.e. ($c^a_L = c^i_L$ and $c^a_D = c^i_D$), we have that:

$$\Delta TW \uparrow \text{ if: } \frac{\theta^1_L}{\theta^2_D} - \frac{\theta^2_L}{\theta^2_D} \rightarrow 0 \left( \frac{N^1_L}{N^2_L} - \frac{N^2_L}{N^2_D} \rightarrow 0 \right)$$

This result shows that CMs with similar borrower/savers members’ ratios achieve more welfare improvement than others once organized in a collective. Interestingly,
this means that the dispersion between CMs’ sizes does not matter ($\gamma_i$ is absent from the equation). The critical feature is the distribution of shares of control between the two groups of members (net borrowers vs. net savers) in CMs. An empirical implication is that collectives regrouping CMs with similar borrower/savers members’ ratios will be more sustainable organizations than others. In this example cases are as follows: Case 1: $N_1^b = N_2^b = \frac{1}{2}; N_1^s = N_2^s = \frac{1}{2}$; Case 2: $N_1^b = \frac{1}{2}; N_2^b = \frac{1}{3}; N_1^s = \frac{1}{2}; N_2^s = \frac{1}{4}$.

### 4.2 Optimal Collectives and the One-man/One-vote Governance Rule

The one-man/one-vote rule affects collective outcome, since the collective formation in CMs 1 and 2 is voted by members. The governance rule of one-man/one-vote used by mutual firms will lead then to a median vote’s outcome ([3]). Let $m_i$ denote the median members in $CM_i$ ($i = 1, 2$). The median voter’s total surplus is defined as follows:

$$x_{m_i} = U_{m_i} + \pi_{m_i}$$

with,
\[ U_{m_i} = \begin{cases} U_b(r_L) & \text{if } m_i \text{ is a borrower} \\ U_s(r_D) & \text{if } m_i \text{ is a saver} \end{cases} \]

\[ \pi_{m_i} = \frac{1}{N_i} \Pi_i(r_L, r_D) \]

Therefore, the formation of the collective will be voted iff:

\[ x_{m_i}(\text{Collective}) > x_{m_i}(\text{Individual}) \]

There are two cases, the borrower and the saver. Both may vote differently.

- **Case 1: \( m_i \) is a borrower-member.**

  Suppose that \( \bar{r}_{m_i} > r_L^*, r_L^{**} \). Hence,

  \[ \Delta x_{m_i} = x_{m_i}(\text{Collective}) - x_{m_i}(\text{Individual}) \]

  \[ = \left[ (\bar{r}_{m_i} - r_L^{**}) + \frac{1}{N_i} \Pi^i(r_L^{**}, r_D^*) \right] - \left[ (\bar{r}_{m_i} - r_L^*) + \frac{1}{N_i} \Pi^i(r_L^*, r_D^*) \right] \]

  \[ = (r_L^* - r_L^{**}) + \frac{1}{N_i} \left( \Pi^i(r_L^{**}, r_D^*) - \Pi^i(r_L^*, r_D^*) \right) \]

  The member will vote for the collective if \( (r_L^* - r_L^{**}) > \frac{1}{N_i} \left( \Pi^i(r_L^*, r_D^*) - \Pi^i(r_L^{**}, r_D^*) \right) \).

- **Case 2: \( m_i \) is a saver-member.**

  Suppose that \( \bar{r}_{m_i} < r_D^*, r_D^{**} \). Hence,

  \[ \Delta x_{m_i} = x_{m_i}(\text{Collective}) - x_{m_i}(\text{Individual}) \]

  \[ = \left[ (r_D^{**} - \bar{r}_{m_i}) + \frac{1}{N_i} \Pi^i(r_L^{**}, r_D^*) \right] - \left[ (r_D^* - \bar{r}_{m_i}) + \frac{1}{N_i} \Pi_i(r_L^*, r_D^*) \right] \]

  \[ = (r_D^{**} - r_D^*) + \frac{1}{N_i} \left( \Pi^i(r_L^{**}, r_D^*) - \Pi^i(r_L^*, r_D^*) \right) \]

  The member will vote for the collective if \( (r_D^{**} - r_D^*) > \frac{1}{N_i} \left( \Pi^i(r_L^*, r_D^*) - \Pi^i(r_L^{**}, r_D^*) \right) \).

The chance of a vote in favor of the collective increases when members of the same bias dominate in both individual CM.

### 4.3 The optimal collective

Let’s introduce the two welfare measures for each CM \( i = 1, 2 \):

\[ \begin{cases} X L_{\text{Individual}}^i = SL^i(r_L^*) + \frac{N_i}{N} \Pi^i(r_L^*, r_D^*) \\ X L_{\text{Collective}}^i = SL^i(r_L^{**}) + \frac{N_i}{N} \Pi^i(r_L^{**}, r_D^*) \end{cases} \]
\[
\begin{align*}
XD^i_{\text{Individual}} &= SD^i(r_D^*) + \frac{N_i^j}{N_i^i} \Pi^a(r_L^*, r_D^*) \\
XD^i_{\text{Collective}} &= SD^i(r_D^{**}) + \frac{N_i^j}{N_i^i} \Pi^a(r_L^{**}, r_D^{**})
\end{align*}
\]

An optimal network (Nash solution) for the two groups of members (borrowers and savers) verifies therefore for each CM
\(i = 1, 2\):

\[
\Delta XL^i = XL^i_{\text{Collective}} - XL^i_{\text{Individual}} > 0
\]

\[
\Delta XD^i = XD^i_{\text{Collective}} - XD^i_{\text{Individual}} > 0
\]

Substituting quantities above by their detailed expressions, we have that,

\[
\frac{1}{N_b^i} \Delta XL^i = \Delta x_{m_i} + \frac{N_j^i}{N_b^i} (r_b^+ - r_L^{**}) \text{ if } m_i \text{ is a borrower-member,}
\]

\[
\frac{1}{N_s^i} \Delta XD^i = \Delta x_{m_i} + \frac{N_j^s}{N_s^i} (r_D^{**} - r_s^-) \text{ if } m_i \text{ is a saver-member,}
\]

with \((i, j) \in \{(1, 2); (2, 1)\}\). This yields that,

if \(m_i\) is a borrower-member, \(\Delta x_{m_i} > 0 \implies \Delta XL^i > 0 \forall i = 1, 2\)

if \(m_i\) is a saver-member, \(\Delta x_{m_i} > 0 \implies \Delta XD^i > 0 \forall i = 1, 2\)

We can also deduce by simple inspection that \(\Delta XD^i > 0\) does not necessarily implies \(\Delta XL^i > 0 \forall i = 1, 2\) and vice versa. Hence, the following result:

**Result 3:** Because of the one-man/one-vote rule, when networks form they are not necessarily a Nash solution for the two groups of members (borrowers and savers) in each CM. However, each voted network is always optimal for the dominating type of members (borrower-type vs. saver-type) in each CM.

## 5 Networks or Hierarchies?

Having considered the conditions under which members will vote for the formation of a collective and the welfare effect of the same, we must now decide upon the organizational forms that the collective may take. Members must decide whether they wish to merge with the partner or simply form an alliance. As noted, the difference between the two is that in the first case the two CM combine property rights creating a single institution, in the second case the CM combine resources and share cash flows but keep legal individuality. The choice set can be represented as follows: \(\{\widehat{\Pi}^i_0 = \max \left[0, \widehat{V}_i - K^0_i\right] - C^0_i, \text{merger}\}\).
We consider the case where (Assumption 2):

- Operational costs are the same among network and hierarchy\(^5\)
- Interest rates are set by market competition

Given Assumptions 1 and 2, the decision rule between the two forms of organization for \(CM_i(i = 1, 2)\) follows from the sign of:

\[
\Delta \Pi^i = \Pi^i_n - \frac{N^i}{N} \Pi^h
\]

with \(N \equiv (N^h_i + N^h_s) = (N^n_i + N^n_s) = (N^1 + N^2)\). That is, the difference for \(CM_i\) \((i = 1, 2)\) in profit is given by the profit in the network minus the profit in the hierarchy, with the latter distributed strictly according to the proportion of members in each CM. We analyze two cases, first a default free loans case and the risky loan case:

The default free loan Case.

Providing Assumption 2, it is easy to show that:

\[
\Pi^a_i = \frac{N^i}{N} \Pi^h \implies \Delta \Pi^i = 0
\]

**Result 4a:** With default risk-free loans (no exogenous uncertainty), CMs are indifferent between network and hierarchy.

The risky loan Case.

To analyze this case we consider the total recovery value of loans as an exogenous random value determined by nature.

- Network (n):

\[
\tilde{\Pi}^a_i = \max \left[ 0, \tilde{V}_i - K^n_i - C^n_i \right]
\]

with

\[
\tilde{V}_i \sim P_i(\cdot)
\]

The distribution probability \(P_i(\cdot)\) is defined over the support \([0, \tilde{V}_i]\)

\[
\tilde{V}_i = \frac{N^i}{N^b} (1 + r_L) L^n(r_L)
\]

\[
K^n_i = \left( \frac{N^i}{N^b} \right) (1 + r_D) D^n(r_D) + \left( \frac{N^i}{N^s} \right) (1 + r_G) G^n(r_L, r_D)
\]

where clearly \(\tilde{V}_i\) represents the face value of assets that corresponds to \(CM_i\) and \(K^n_i\) the face value of liabilities of \(CM_i\).

\(^5\)Transaction costs economics theorists would argue that governance costs of hybrids will be lower than those of mergers for low levels of contractual risk but the first display a higher slope as contractual risk increases than the second. (e.g. [7], [8]) Further, gains in production costs will be higher for mergers than for hybrids. The interaction of these three elements determines which form dominates. To establish cateris paribus conditions, and preclude a bias that may favour one or the other or the basis of costs we assume identical costs gains. That is, in transaction costs terms, we are at the intersection of the efficient frontier for the hybrid and hierarchy forms, where costs are identical.
• **Hierarchy (h):**

\[ \tilde{\Pi}^h = \max \left[ 0, \tilde{V}^h - K_i^h \right] - C^h \]

with

\[ \tilde{V}^h \sim P(.) \text{ with } \tilde{V}^h = \sum_i \tilde{V}_i \]

The distribution probability \( P(.) \) is defined over the support \([0, \nabla^h] \]

\[ \nabla^h = (1 + r_L)L^h(r_L) \]

\[ K_i^h = (1 + r_D)D^h(r_D) + (1 + r_G)G^h(r_L, r_D) \]

where assets and liabilities have been pooled.

Remark that given Assumption 1:

\[ V^h = \sum_i V_i \]

\[ K_i = \sum_i K_i^n \]

\[ C_i = \sum_i C_i^n \text{ and } C_i^n = \left( \frac{N_i}{N} \right) C_i^h \]

which in turn implies,

\[
E[\Delta \Pi] = E[\Pi_1^n] - \frac{N_i}{N} E[\Pi_2^h] \\
= \left\{ E \left[ \tilde{V}_i \mid \tilde{V}_i > K_i^n \right] - K_i^n \left[ 1 - P_i(K_i^n) \right] \right\} \\
= \left( \frac{N_i}{N} \right) \left\{ E \left[ \sum_i \tilde{V}_i \mid \sum_i \tilde{V}_i > \sum_i K_i^n \right] - \sum_i K_i^n \left[ 1 - P_i(\sum_i K_i^n) \right] \right\}
\]

**Result 4.b.** Suppose that \( \frac{N_i^1}{N_1^2} = \frac{N_i^2}{N_2^2} \) (identical CMs in terms of member bias). Then,

\[
E[\Delta \Pi] > 0 \text{ if } \tilde{V}_i \succeq SD \frac{N_i}{N} \sum_i \tilde{V}_i
\]

(SD: Stochastic Dominance). Hence, where assets (loans) are good quality because of the exogenous default risk, network is more suitable than hierarchy if it permits to CM with low exposure to default risk (high quality assets) to protect their residual claim better than in hierarchy where assets are pooled together. This means that networks will be more present in economies where assets (loans’ portfolios) are highly specific. A typical example would be the case of CMs maintaining a high proximity with their members such as rural cooperatives. We illustrate the situation with a numerical example and a graph. In that example we use:

\[ \tilde{V}_i = \bar{V}_i \exp (-|\varepsilon_i|), \varepsilon_i \sim N(0, \sigma_i) \forall_i = 1, 2 \]

\[ \bar{V}_i = 100 \forall_i = 1, 2 \]

\[ K_i^n = 60 \forall_i = 1, 2 \]

\[ \sigma_1 = 2.5 \text{ (maintained constant); } \sigma_2 \in [0, 5] \]

In this context \( \frac{\sigma_1}{\sigma_2} \uparrow \Rightarrow \tilde{V}_1 \succeq SD \frac{N_i}{N} \sum_i \tilde{V}_i \). Assets quality motivation for CM 1 is high, and, hence, network is more suitable than hierarchy.
6 Conclusion

This paper presents a model of network formation for credit mutuals (CM). The importance of the subject resides in the fact that CM as well as many other types of mutual organizations tend often to integrate into network-like structures. Sometimes these structures are so imposing that they have become, when taken as a network, the primary, or one of the primary, financial institutions in the country. However, little is known about the factors that influence their formation and stability. The model presented in this paper attempts to address precisely this gap.

We present the model in three steps. In the first step we develop a simple model of a credit mutual that maximizes member monetary and non-monetary surplus. Then we define the basic structure of a network that separates it from a conglomerate with subsidiaries, and, based on that definition, build a model of a network by assuming absence of default. This basic model allows us to draw a number of predictions about the factors that influence the formation of network. Amongst the more interesting ones we may cite that operational efficiency is not the (only) reason for networks to be desirable. Other factors, such as diversifying the mixture of membership (savers vs. borrower members) may also have an influence. Further, members may draw benefits from networking from benefiting from additional monetary and non-monetary surpluses. In the third step, we incorporate default risk to the model and derive a new
set of equilibrium conditions. These conditions suggest that increases in (expected) default risk will discourage network formation and that, in exiting networks, CM with high loan default rates will tend to be marginalized from the decision process.

From the policy point of view we can draw the following conclusions. First, policy makers attempting to encourage network formation as a means to accelerate CM growth—to provide micro financial services to poorer sectors of the population—can do that by exploiting several venues, such as improving on operational efficiency through networks and mixing different types of CM in terms of membership. Further, periods of high loan default rates are bad periods to encourage network formation. Further, that either in existing networks or new networks mechanisms should be included in the governance rules that reduce the influence of high default rate CM, perhaps by suspending or reducing temporarily their voting right.

References


