# Information Communication in Financial Networks*

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**Abstract**

Empirical evidence shows word-of-mouth communication has real effects on trading patterns in financial markets. On top of that, the social network through which the information conveys also plays a crucial role. We propose a framework that allows informed traders to directly and truthfully communicate information in four major social interaction structures represented by circle, tree, star and complete network graphs. In particular, we generalize à la Kyle (1985)'s strategical noisy rational expectation model by incorporating directed information transmission. We highlight that individual exploits information distinctly in different networks, and individual's influence on market price relies on the network type and position in the network. Despite of the distinction on individual trading behavior and influential power on price, market trading patterns are similar irrespective of the network structures. Trading volume, price volatility and the information efficiency of price are all higher in networks than their counterparts in conventional models without networks.

## 1 Introduction

Information communication among agents in social environment is an important and pervasive feature in financial decision-making. However, in standard market microstructure

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literature, heterogeneous traders are assumed to make decisions in isolation by exploiting their own private information and market prices. When they take into account their competitors' strategic decisions, they never consider word-of-mouth communication or any other exchange means regarding asset information even when they have personal contact with their competitors. In this sense, every trader lives in a disconnected island alone.

The general objective of this paper is to incorporate the information transmission in different network forms into financial market. We try to study not only the influence of information communication in financial decision, but also the role played by the social network through which the communication takes place. In particular, we propose a strategical noisy rational expectation model à la Kyle (1985) in which informed traders (insiders) engage in *direct* and *truthful* information transmission in four major social interaction structures, represented by circle, tree, star and complete network graphs. We model the network of insiders as a set of linked nodes. Specifically, in the circle network, all insiders are physically located clockwise therefore everyone has one closest neighbor to her right side and one to her left side. In tree network, one insider on top is connected to a bunch of successors and each successor is in turn connected to her successors. In the star network, a special case of tree network, one insider is located in a central position and is surrounded by all other disjointed insiders. In complete network, each insider is connected to all other insiders in the network. In regard to the modeling of information flow we try to accommodate the main features of information transmission in virtually all financial markets by most natural and conventional way. In circle network, we assume every insider receives information from her closest right side neighbor and then transmits her private signal to her closest left side neighbor. In tree network, every insider receives information from her unique predecessor and delivers her private information to her direct successors. In star network, peripheral insiders unilaterally receive information from the central insider. And in complete network, each insider receives information from all other insiders in the network. This modeling successfully covers diverse forms of information flow: one-way directed information flow in circle, tree and star network, and two-way flow in complete network, and meanwhile facilitates closed-form solutions as well.

In our model, heterogeneity in individual trading behavior might arise due to insiders' asymmetric positions and communication forms in the network. Two questions arise naturally: how do insiders exploit their own private signals and their received information? Whose information has larger influence on market price? The first one is the central theme of social learning theory which studies how individual rationally responds to group behavior when others base their decisions on their information. The second one is also one of the crucial concerns in financial theory which illustrates how market price aggregates and reveals multiple information. We demonstrate that the answers to these two questions mainly hinge on two factors: signal precision and network position. For the former question, traditional thinking is that people will rely more on the signal which is more precise if all signals have equal correlations with fundamental value of assets. But this result might be reversed in certain network structures. For example, in star network, although peripheral insiders receive superior information from center insider, they still count more on there own lower quality signals for the reason that the center insider's signal is utilized by other peripheral insiders. For the latter question, insider's information influential power on market prices highly depends on the position in network hierarchy. The higher level the insider is in the hierarchy, the more influential the insider's signal is. Specifically in tree network, each insider has larger influence than her successor, and there influential power ratio increases with the number of successors and approaches 2 when that number goes to infinity.

Despite the distinction of information utilization and price influence in individual level, we highlight that other trading patterns summarized as expected volume and price volatility are similar in market level irrespective of network structures. We compare them in different network forms to those in benchmark model that assumes away the role of interpersonal and interactive communication. We establish that trading volume and price volatility are all higher in networks than warranted by benchmark. The enormity of trading volume and price volatility has long been a puzzle since first stated by Shiller (1981) and LeRoy and Porter (1981). Both the rational and behavioral approaches to finance have made progress in understanding this puzzle over the last two decades. Behavior finance resorts to irrational belief and non-standard preference to explain the puzzle. In Wang (1998) and Odean (1998)), overconfidence is exploited to address the volume and volatility phenomenon. Barberis and Huang (2001) propose a utility function incorporating lose aversion to explain the puzzle. On the rational side, Grossman and Stiglitz (1980), Kyle (1985) and Admati and Pfleiderer (1988) all point out liquidity trading contribute to the enormous trading activity and price fluctuation. But attributing the volume and volatility phenomenon to behavior story or liquidity trading as the primary sources is unappealing both theoretically and empirically. Our paper here suggests that network might be a plausible way of thinking about the puzzle.

The remainder of this paper is organized as follows. Section 2 provides literature review of the effects of social interaction and word-of-mouth communication in financial markets as well as in other economic activities. In section 3 we propose the benchmark model in which every informed trader acts strategically in isolation and we introduce useful market statistics as guidelines for future assessment of social interaction. Information transmission and insider trading in circle, tree, star and complete networks are studied in section 4,5,6,7 respectively. We demonstrate the associated dissimilarity in individual trading behavior and similarity in market trading patterns. Section 6 concludes. All analytical proofs are relegated to the appendix.

## 2 Literature Review

Over the last two decades, the empirical works on information communication in network have been extensively conducted in two strands. The first strand documents the interpersonal communication is relevant and even critical in financial decision-making. Hong, Kubik, and Stein (2004) test the hypothesis that the decisions of U.S. households to invest in the stock market or not are influenced by their social interaction with neighbors. The data show those investors who they characterize as social are about 4% more likely to invest in the stock market than those who they characterize as non-social, controlling for important background factors such as wealth and education. Their estimate of the increase in probability of investing for social households is even larger in those states where proportion of the population investing is the highest. Shiller (2000) provides a comprehensive and in-depth investigation of social influence and world-of-mouth communication among traders in financial markets and argues that these factors, among others, contribute to the irrational exuberance of late nineties. In particular, Shiller highlights that interpersonal communication is more influential than traditional media. The recent popularity of the Internet as a medium of financial markets discussion attracts economists' attention on another way of information communication. Wysocki (1999) studies the 50 firms with the highest message posting and reports that message posting forecasts next-day trading volume and next-day abnormal stock returns. Antweiler and Frank (2004a, 2004b) argue that people put their money where their mouth is. They find that stock message boards have strong predictability for volatility. High message posting is associated with greater next day volatility.

Another strand of the empirical papers confirms the essential role played by network structure in financial decision-making. They argue that social interaction is usually subject to factors as geographical distance, social relationship and there is evidence that the proximity influences investors' portfolio choices. For instance, by examining the network of traders on the floor of the options exchange, Baker (1984) developed a sociological study of markets and demonstrated that the social structural patterns dramatically affected the direction and magnitude of option price volatility. Kelly and O'Grada (2000) investigate the behavior of Irish depositors in a New York bank during the panics of 1854 and 1857. The social networks are determined by the place of origin in Ireland and neighborhood in New York. The paper shows that social network is the main factor in the determination of depositors' behaviors. Coval and Moskowitz (1999) demonstrate that U.S. portfolio managers exhibit a strong preference for locally headquarter companies in their investment. Hong, Kubik, and Stein (2002) show that, controlling for what fund managers in other cities are doing, fund managers are more likely to hold or trade stocks if their neighbors do so. And managers' decisions are lowly correlated if they live far from each other. They find that fund managers' holdings of a particular stock as a percentage of their total portfolios increase by roughly 0.2% when fund managers at different firms in the same city increase their holdings in the same stock by 1%.

Prompted by the empirical findings presented above, a few papers build analytical models to study the interplay between rational investors in financial network, Ozsoylev (2003) and Ozsoylev (2005) study the existence and properties of equilibrium price by introducing social interaction into the competitive noisy rational expectation model *à la* Hellwig (1980). On top of the information conveyed through the price system, every asymmetrically informed trader infers additional information by observing the assets demand of her neighbor physically located on her right side, on her upper side or in the central. The corresponding social networks are circle, tree or star respectively. Our paper is close to Ozsoylev (2003, 2005) regarding the problem studied and network structure employed. The main difference between Ozsoylev (2003, 2005) and our paper is the model we adopt. Kyle (1985)' model captures the intuition of strategically trading that Hellwig (1980) fails to considerate. Moreover, leaning in Ozsoylev (2003) takes place through observation of other's demands instead of signals assumed in our paper. Our paper yields some important implications sharply contrast to Ozsoylev (2003). For instance, in our circle network model, we find a closed form linear equilibrium which can not be sustained in Ozsoylev (2003). Furthermore, Ozsoylev (2003) demonstrates that price taking behavior may not be justified by a large economy with tree or star network because the upper and central investor's behavior has a larger or even infinite impact on equilibrium price. In our model, we conclude any investor's impact on price is limited.

Contrast to the one-way directed information transmission, Colla and Mele (2004) investigate a two-way information flow model. They develop a dynamic trading model with informed traders engaging in truthful information sharing in a circle network about the terminal value of an asset. In particular, before trade begins, every informed trader has an one-shot information exchange through conversation with her neighbors located on both sides, and all of them are located around a circle. Informed traders then participate in a sequence of batch auctions *à la* Foster and Viswanathan (1996), they not only forecast the asset's fundamental value but also forecast their competitors' forecast. Their motivation for information sharing is justified by the fact that under a wide range of conditions on initial beliefs heterogeneity and the market structure summarized by the number of investors and batch auctions, traders' optimal and symmetric choice of the size of conversation group entails gains. The explanation for their finding is very similar to those in the voluntary disclosure literature (Diamond, 1985). On the one hand, information sharing reduces traders' monopolistic power. On the other hand, it improves the quality of traders' forecast about the asset's fundamental value. When the initial correlation between investors' signals is high enough, the losses generated by the first effect are smaller than the gains associated with the second effect.

Sequential trade model, the other workhorse in market microstructure theory, implicitly assumes a linear social network. Each successive trader infers additional information by observing predecessors' demand. (Glosten and Milgrom, 1985). Because this similarity in the social learning structure, information cascade models (Banerjee, 1992; Bikhchandani et al., 1992) have been connected to sequential trade model to generate some interesting herding behavior in financial markets. (Avery and Zemsky, 1995; Chari and Kehoe[1], 2004). Chamley (2004) provide extensive literature reviews on social learning theory. Learning in network is an important component.

The role of interpersonal and interactive communication through social network in decision making has long been recognized in other fields of economic activities. Neighbors are introduced in models of learning from others' action by Allen (1982a, 1982b). The word-of-mouth communication through social networks have been shown to play major role in technology adoption, their different structures can determine the technology conformity or diversity in the long run. (Ellison and Fudenberg 1995; Bala and Goyal 1998, 2000, 2001).

[1] Chari and Kehoe (2004) allow the agents to determine action time.

The neighbor and peer effects in labor markets have been studies extensively. Jackson (2000) surveys the progress of network formation.

# 3 Benchmark: Insider Trading without Information Transmission

Our benchmark model is a natural extension of Kyle (1985). Other than notational differences, the only change made to Kyle's seminal batch auction framework, also known as strategic noisy rational expectation model, is that there are $n$ informed traders (insiders) and their private signals of the asset intrinsic value are noisy. In the benchmark no social network has been established in the financial markets, and every informed trader lives in a disconnected island and trades in the conventional manner. We present it in this section for two purposes. First, it will be easier for readers to compare the approach and implication of standard strategically trading model with those of ours in which financial networks are introduced and their effects are studied. Secondly, Even though our financial network models accommodate the benchmark model in that all traders return to the isolating world once the underlying social network graphs are replaced by disjointed nodes, the equilibrium results derived in the benchmark model cannot be directly followed by the same treatment.

This is a one-period model in which $n$ $(n > 1)$ risk-neutral, privately and diversely informed traders (the insiders) and noise traders submit market orders simultaneously to a risk-neutral market maker. Trading takes place at time 1 and the assets are liquidated at time 2. The fundamental value of the single risky asset $\tilde{v}$ is normally distributed $\boldsymbol{N}\left(\bar{v}, \sigma_v^2\right)$ where $\bar{v}$ is assumed to equal to 0; this simplifies notation without affecting the propositions. Prior to trading, at time 0 each risk-neutral insider $i$ is endowed with a private signal $\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i$ where $\tilde{\varepsilon}_i$ is normally and identically distributed $\boldsymbol{N}\left(0, \sigma_\varepsilon^2\right)$ for $i = 1, \cdots, n$. Noise traders, who trade for unmodeled idiosyncratic or liquidity reasons, together submit an exogenous random quantity $\tilde{u}$ which is normally distributed $\boldsymbol{N}\left(0, \sigma_u^2\right)$. The random variable $\tilde{v}$, $\tilde{\varepsilon}_i$ and $\tilde{u}$ and assumed to be mutually independent for all $i$. A competitive risk-neutral market maker absorbs the net demand of the other traders and sets the price such that he expects to earn zero profit. The zero-profit condition will be satisfied if it is determined as the outcome of a Bertrand auction for the order flow between at least two market makers.

Insider $i$'s trading strategy is given by a measurable function $X_i : \mathbb{R} \rightarrow \mathbb{R}$, determining her market order as a function of her information set $\tilde{I}_i$. For a given strategy, let $\tilde{x}_i = X_i\left(\tilde{I}_i\right)$. A strategy profile $(X_1, \cdots, X_n)$ determines order flow $\tilde{\omega} = \sum_{i=1}^{n} \tilde{x}_i + \tilde{u}$. Market maker's pricing rule is given by a measurable function $P : \mathbb{R} \to \mathbb{R}_+$. Given $(P, X_1, \cdots, X_n)$, define $\tilde{p} = P(\tilde{\omega})$ and let $\tilde{\pi}_i = (\tilde{v} - \tilde{p})\, \tilde{x}_i$ denote the resulting profit for insider $i$.

In the benchmark no traders engage in information transmission. Based on her own information and belief, each insider acts strategically by taking into account the fact that her optimal demand $\tilde{x}_i$, as well as others' decisions, in the camouflage of the noise traders, will influence the risky asset price and her profit. Market maker attempts to infer the private information from the order flow and sets price as efficiently as possible in order to rule out the profit opportunities of the insiders.

We call above trading structure or market arrangement as Kyle's market. In the following discussion, we give equilibrium strategy and related parameters different symbol subscripts as an indicator of different networks. In the benchmark no subscript is used. Because of its game-theoretical flavor, we define the equilibrium as follows. Here $\boldsymbol{E}[\cdot|\cdot]$ denotes conditional expectation.

**Definition 1** *A Nash Equilibrium in the Kyle's market without information transmission is $\{X_1, \cdots, X_n, P\}$, the insiders' trading strategy profile and the market maker's pricing rule, such that the following conditions hold:*

*1. Profit maximization: for insider $i$'s any alternative trading strategy $X_i^{'}$*

$$\begin{aligned} &\boldsymbol{E}\left[\left(\tilde{v} - P\left(\sum_{j\neq i} \tilde{x}_j + X_i\left(\tilde{I}_i\right) + \tilde{u}\right)\right) X_i\left(\tilde{I}_i\right) \middle| \tilde{I}_i = I_i\right] \\ \geq\ &\boldsymbol{E}\left[\left(\tilde{v} - P\left(\sum_{j\neq i} \tilde{x}_j + X_i^{'}\left(\tilde{I}_i\right) + \tilde{u}\right)\right) X_i^{'}\left(\tilde{I}_i\right) \middle| \tilde{I}_i = I_i\right] \end{aligned} \tag{3.1}$$

*where $\tilde{I}_i = (\tilde{s}_i)$ is insider $i$'s information set for $i = 1, \cdots, n$.*

*2. Market semi-strong efficiency: the pricing rule $P$ satisfies*

$$P(\tilde{\omega}) = \boldsymbol{E}\left[\tilde{v} \middle| \tilde{\omega} = \sum_{i=1}^{n} \tilde{x}_i + \tilde{u}\right] \tag{3.2}$$

It is well known that the benchmark model has a unique linear equilibrium as shown below:

**Theorem 1** *Without information transmission, there exists a unique linear equilibrium in the Kyle's market in which* $X$ *and* $P$ *are given by*

$$X(\tilde{s}_i) = \beta \tilde{s}_i \tag{3.3}$$

$$P(\tilde{\omega}) = \lambda \left( \sum_{i=1}^{n} \tilde{x}_i + \tilde{u} \right) \tag{3.4}$$

*where the trading intensity parameter* $\beta$ *and liquidity parameter* $\lambda$ *are given by*

$$\beta = \frac{\sigma_u}{\sqrt{n\sigma_v^2 + n\sigma_\varepsilon^2}} \tag{3.5}$$

$$\lambda = \frac{\sigma_v^2}{\sigma_u} \frac{\sqrt{n\sigma_v^2 + n\sigma_\varepsilon^2}}{(1+n)\,\sigma_v^2 + 2\sigma_\varepsilon^2} \tag{3.6}$$

*The second order condition* $\lambda > 0$ *is satisfied.*

In (3.4) liquidity parameter $\lambda$ determines the price change of an additional order. When $\lambda$ is low, an additional order will not cause a large price change, the market is thus very liquid. Consequently, the informed is induced to trade more aggressively. It is very important to see that $\lambda$ is non-monotonic in $n$.

$$sign\left(\frac{\partial \lambda}{\partial n}\right) = sign\left(\frac{1-n}{2\sqrt{n}}\sigma_v^2 + 2\sigma_\varepsilon^2\right)$$

A direct corollary is that when we take $n = 1$ and $\sigma_\varepsilon^2 = 0$, then we obtain the classical results in Kyle (1985).

$$\beta = \frac{1}{2\lambda} = \frac{\sigma_u}{\sigma_v}$$

$$\lambda = \frac{\sigma_v}{2\sigma_u}$$

In addition to trading intensity and market liquidity, we introduce several market statistics, including expected volume, price volatility, price efficiency, and expected profit. They are useful in the following comparison.

Following Admati and Pfleiderer (1988), the total trading volume, denoted by $\widetilde{Vol}$, is defined by

$$\widetilde{Vol} = \frac{1}{2}\left( \sum_{i=1}^{n} |\tilde{x}_i| + |\tilde{u}| + |\tilde{\omega}| \right)$$

Given that $\tilde{x}_i, \tilde{u}$ and $\tilde{\omega} = \sum_{i=1}^{n} \tilde{x}_i + \tilde{u}$ are normally distributed with zero means, simple calculations, using the statistical formula in Appendix A, yield the expected trading volume as

$$\begin{aligned}\boldsymbol{E}\left(\widetilde{Vol}\right) &= \frac{1}{\sqrt{2\pi}}\left(n\beta\sqrt{\sigma_v^2+\sigma_\varepsilon^2}+\sigma_u+\sqrt{n^2\beta^2\sigma_v^2+n\beta^2\sigma_\varepsilon^2+\sigma_u^2}\right)\\ &= \frac{\sigma_u}{\sqrt{2\pi}}\left(1+\sqrt{n}+\sqrt{\frac{(1+n)\,\sigma_v^2+2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}}\right)\end{aligned}$$

Price volatility is measured by the variance of equilibrium price and it is

$$\boldsymbol{var}\,(\tilde{p}) = (\beta\lambda)^2\left(n^2\sigma_v^2+n\sigma_\varepsilon^2\right)+\lambda^2\sigma_u^2 = \frac{n\sigma_v^4}{(1+n)\,\sigma_v^2+2\sigma_\varepsilon^2}$$

In the literature, the information efficiency (informativeness) of price is either measured by the posterior precision of $\tilde{v}$ or by the residual variance of $\tilde{v}$, conditional on the equilibrium price $\tilde{p}$ (or the order flow $\tilde{\omega}$). It is easy to see that they are informatively equivalent. We adopt the first and obtain the following from an application of the projection theorem in Appendix A,

$$[\boldsymbol{var}\,(\tilde{v}|\tilde{p})]^{-1} = \frac{(1+n)\,\sigma_v^2+2\sigma_\varepsilon^2}{\sigma_v^4+2\sigma_v^2\sigma_\varepsilon^2}$$

It is worth noting that the variance of noise trading does not affect price volatility and price efficiency, because insiders scale up their trading intensity in response to an increase in the amount of noise trading.

In equilibrium, the ex ante expected profit, denoted by $\Pi$, of insider $i$, is

$$\Pi_i = \frac{\sigma_v^2\sigma_u}{(1+n)\,\sigma_v^2+2\sigma_\varepsilon^2}\sqrt{\frac{\sigma_v^2+\sigma_\varepsilon^2}{n}}$$

In the following models, we will incorporate various social networks into the benchmark, and we will investigate the existence of equilibrium and corresponding market statistics.

# 4 Insider Trading in Circle Network

In this section we study the information transmission and insider trading in financial circle network. We first describe the structure of circle network and the direction of information diffusion, and then we discuss their economic interpretation.

The environment of this model is identical to benchmark except that $n$ informed traders are ordered clockwise, as to say that trader $i$ has trader $i+1$ to her left and trader $i-1$ to her right, and they engage in transmitting information in a special fashion specified below. The graph in Figure 1 is a symbolic representation of a network. It implies an abstraction of the reality so it can be simplified as a set of linked nodes. We call it circle only for descriptive convenience. Circle is a special form of *cycle* in graph theory, which can be defined casually as a chain where the initial and terminal node is the same and that does not use the same link more than once[2].

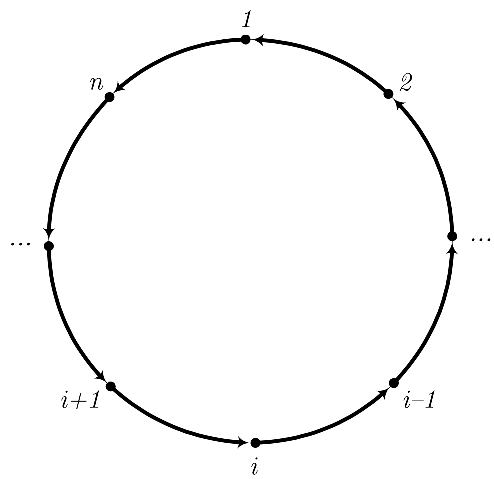


Figure 1: **Information transmission in circle network.** The set of linked nodes represents social interaction among insiders. Arrows indicate the direction of information transmission. For all $i = 1, ..., n$, insider $i$ has a private signal and additional received signal about the fundamental value of an asset from insider $i + 1$.

Between time 0 and 1, in addition to her private owned signal, each insider also receives a signal from her closest left side "neighbor". And at the same time, she transmits her own signal to her closest right side "neighbor". In Figure 1, arrows indicate the *one-way* direction of information transmission. In the language of graph theory, our circle network

[2]More rigorously, Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a graph with the set of nodes (vertices) $\mathcal{N}$ and the set of links (edges) $\mathcal{L}$. A path from a vertex $v_1$ to a vertex $v_k$ is an alternating sequence of vertices and edges, $(v_1, v_2, \cdots, v_k), v_i \in \mathcal{V}$, all the vertices in the sequence are distinct and successive vertices $v_i$ and $v_{i+1}$ are endpoints of the intermediate edge $(v_i, v_{i+1}) \in \mathcal{L}$. If we only allow the first and last vertices to coincide, we call the resulting closed path a cycle. A graph with no cycles is called a tree.

can be called a *directed cycle*, or more formally, a *circuit*[3]. In another word, insider $i$'s information set $I_i$ includes both her private signal $s_i$ and the signal $s_{i+1}$ transmitted from insider $i+1$.Insider $i$ will also transmits her signal $s_{i+1}$ to trader $i-1$, and so on. We assume that all insiders transmit *truthful* information in that nobody distorts her signal[4]. Our model therefore rules out strategic information transmission and information-based price manipulation. Trade occurs in period 1.

The circle network can be thought of as an unusual clock with $n$ numbered dial. For example, 5 traders after $(n-2)^{th}$ trader is the $3^{rd}$ trader, in the following calculation, we use modular arithmetic with modulus $n$ and simply denote it as $\pmod{n}$ when necessary.

Obviously, our modeling of the information transmission in the circle network differs from that in Ozsoylev (2003) and Colla and Mele (2004). The former assumes traders believe action is louder than word, so every informed trader observes her right side neighbor's assets demand rather than the signal itself. The latter assumes that every informed trader exchanges her information for that of her both sides' neighbors. So they focus on information sharing instead of information transmission. For example, when trader $i$ shares information with $i-2$, $i-1$ and $i+1$, $i+2$, then she has $I_i = (\tilde{s}_{i-2}, \tilde{s}_{i-1}, \tilde{s}_i, \tilde{s}_{i+1}, \tilde{s}_{i+2})$ $\pmod{n}$ for $i = 1, \cdots, n$. In the following we will demonstrate our modeling choice yields more fruitful results.

Although the circle network considered is something of a modeling contrivance, we do not think that it is too unrealistic. In fact, it resembles quite closely some important features of information transmission in virtually every financial markets.

From now on, subscript $\circ$ is used to denote the circle network.

**Definition 2** *A Nash Equilibrium in Kyle's market with information transmission in financial circle network represented above is similarly defined as Definition 1 except that for* $i = 1, \cdots, n$, $\tilde{I}_i = (\tilde{s}_i, \tilde{s}_{i+1})$

When an insider's information set is multi-dimensional and the received information is other trader's private signal, a distinctive feature of insider's profit maximization problem is that her information set overlaps with other insiders' information sets. We have to deal with the covariance of related information carefully. For example, consider insiders $i$ and

[3] Circuit is a path where the initial and terminal node correspond and is a cycle where all the links are traveled in the same direction.

[4] If every insider has an i.i.d. noise term to her to her received information, we believe that all qualitative results of this paper will not change, while the calculation will be quite cumbersome.

$i+1$, the covariance of $(\tilde{s}_i, \tilde{s}_{i+1})$ and $(\tilde{s}_{i+1}, \tilde{s}_{i+2})$ is non-zero. Fortuitously, a unique linear equilibrium is conjectured and verified.

**Theorem 2** *With information transmission in financial circle network represented above, there exists a unique linear equilibrium given by*

$$X_{\circ i}\left(\tilde{s}_i, \tilde{\delta}_i\right) = \beta_\circ \tilde{s}_i + \gamma_\circ \tilde{s}_{i+1} \tag{4.1}$$

$$P_\circ(\tilde{\omega}) = \lambda_\circ \left(\sum_{i=1}^{n} \tilde{x}_{\circ i} + \tilde{u}\right) \tag{4.2}$$

*where the trading intensity parameters $\beta_\circ$ and $\gamma_\circ$, and liquidity parameter $\lambda_\circ$ are given by*

$$\beta_\circ = \frac{\sigma_u}{\sqrt{n(4\sigma_v^2 + 2\sigma_\varepsilon^2)}} \tag{4.3}$$

$$\gamma_\circ = \frac{\sigma_u}{\sqrt{n(4\sigma_v^2 + 2\sigma_\varepsilon^2)}} = \beta_\circ \tag{4.4}$$

$$\lambda_\circ = \frac{\sigma_v^2 \sqrt{n(\sigma_v^2 + \frac{\sigma_\varepsilon^2}{2})}}{\sigma_u \left[(n+1)\sigma_v^2 + \frac{3\sigma_\varepsilon^2}{2}\right]} \tag{4.5}$$

*The second order condition $\lambda_\circ > 0$ is satisfied.*

Our first proposition is about the expected trading volume.

**Proposition 1** *Expected trading volume in circle network is higher than that in benchmark.*

$$\begin{aligned}
\boldsymbol{E}\left(\widetilde{Vol}_\circ\right) &= \frac{1}{\sqrt{2\pi}} \left\{ n\sqrt{(\beta_\circ + \gamma_\circ)^2 \sigma_v^2 + \left(\beta_\circ^2 + \gamma_\circ^2\right) \sigma_\varepsilon^2} + \sigma_u + \sqrt{(\beta_\circ + \gamma_\circ)^2 (n^2\sigma_v^2 + n\sigma_\varepsilon^2) + \sigma_u^2} \right\} \\
&= \frac{\sigma_u}{\sqrt{2\pi}} \left[ \sqrt{n} + 1 + \sqrt{\frac{(2n+2)\sigma_v^2 + 3\sigma_\varepsilon^2}{2\sigma_v^2 + \sigma_\varepsilon^2}} \right] \\
&> \boldsymbol{E}\left(\widetilde{Vol}\right)
\end{aligned}$$

Insiders trade more aggressively after information transmission simply because they can exploit more diverse information.

In practice, the tremendous trading volume in financial markets is a challenge to the "no-trade theorem" developed by Milgrom and Stoky (1985). Several motives have been intensively explored in the literature. The research lines in competitive and strategic noisy rational expectation models, pioneered by Grossman and Stiglitz (1980), and Kyle (1985) respectively, demonstrate that private information and liquidity or noise trading are the major motives for trade. More recently, heterogeneous prior beliefs has been proposed as another significant motive for trade. The enormous volume in Harris and Raviv (1993) arise from different priors about an asset's intrinsic value while in Wang (1998) and Odean (1998) is due to non-concordant prior beliefs about the relationship between the signal and the asset value.

Our resort to social interaction effect in explaining the volume phenomenon is brand new and has some advantage. Clearly, for a fixed amount of noise trading, incorporating information transmission in circle network generates higher trading volume than warranted by benchmark.

The empirical studies document the positive correlation between trading volume and contemporaneous price volatility or absolute price changes. (Karpoff, 1987; Jain and Joh, 1988; among others) Our model predicts this evidence. We also demonstrate that the informativeness of prices is higher.

**Proposition 2** *Price volatility and price efficiency are higher in circle network relative to benchmark.*

$$\boldsymbol{var}\left(\tilde{p}_{\circ}\right) = \frac{n\sigma_v^4}{(1+n)\sigma_v^2+\frac{3}{2}\sigma_\varepsilon^2} > \boldsymbol{var}\left(\tilde{p}\right) \tag{4.6}$$

$$\left[\boldsymbol{var}\left(\tilde{v}|\tilde{p}_{\circ}\right)\right]^{-1} = \frac{(1+n)\,\sigma_v^2+\frac{3}{2}\sigma_\varepsilon^2}{\sigma_v^4+\frac{3}{2}\sigma_v^2\sigma_\varepsilon^2} > \left[\boldsymbol{var}\left(\tilde{v}|\tilde{p}\right)\right]^{-1} \tag{4.7}$$

We can see $\sigma_u^2$ doesn't appears in these equations, The size of noise trader does not affect the price volatility and price efficiency. The reason is the same as before; insiders adjust their trades in response to changes of noise trading.

This proposition tells that the price volatility in circle network exceeds that in benchmark imposing social isolation. As we know, enormous volatility ratio of price to fundamentals can't be empirically justified by using aggregate U.S stock market data. We cited some rational and behavior approaches contributing in understanding this puzzle. This proposition proposes another plausible justification for this volatility puzzle: network effect.

# 5 Insider Trading in Tree Network

In this section we consider a new financial network in which the social interaction is represented by a tree. A tree graph which represents the hierarchical nature of social networks is a most natural example of graph without circles. And also it brings asymmetry into the social network, hence heterogeneity into the financial market economy. In graph theory, a graph with no cycles is called acyclic graph, or vividly called a forest. A tree is a connected acyclic graph. All acyclic graphs can be reduced to trees if we make the following two assumptions. (1) Each agent has at most one uphill neighbor. (2)There are no disjoint subgraphs[5].

For the sake of the tractability, we start to analyze a simple but standard tree which contains "root", "nodes" and "leaves". The root is the starting point of the network which has no predecessor. It is the also parent of nodes. A node is a parent of certain number leaves and is the direct successor of the root. The leaves are at the lowest level in the hierarchy and they have no children in the network. We assume the root has $k$ successive nodes, and each node has $k$ successive leaves. In total, there are $n = 1 + k + k^2$ insiders in the network. We denote the root agent as agent 0, the nodes as agents 1 through $k$, and the leaves agents in the family of node agent $i$ as agent $ik + j$. Here $i = 1, \cdots, k$ and $j = 1, \cdots, k$ .The root will transmit his private signal to his children nodes, each node will transmit his private signal to his children leaves. Figure 2 depicts network structure and the flow of information transmission.

We still assume away the strategic information transmission or information-based price manipulation. Nodes and leaves insiders receive *truthful* private signals from their parents. We impose this strong assumption for consistent consideration.

About the signal, we keep the same assumption as in benchmark. Each insider $i$ possesses a private signal $\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i$, where $\tilde{v}$ is normally distributed $\boldsymbol{N}\left(0, \sigma_v^2\right)$ and $\tilde{\varepsilon}_i$ is normally and identically distributed $\boldsymbol{N}\left(0, \sigma_\varepsilon^2\right)$ for $i = 1, \cdots, n$.

**Definition 3** *A Nash Equilibrium in Kyle's market with information transmission in financial tree network represented above is similarly defined as Definition 1 except that* $\tilde{I}_0 = (\tilde{s}_0)$ , $\tilde{I}_i = (\tilde{s}_0, \tilde{s}_i)$ *for* $i = 1, \cdots, k$,*and* $\tilde{I}_{ik+j} = (\tilde{s}_i, \tilde{s}_{ik+j})$ *for* $i = 1, \cdots, k$ *and* $j = 1, \cdots, k$.

[5] No disjoint subgraphs means there do not exist two disjoint subsets of agents, say $F_1, F_2$,such that no agent in $F_1$ is observed by an agent in $F_2$, and no agent in $F_2$ is observed in $F_1$.

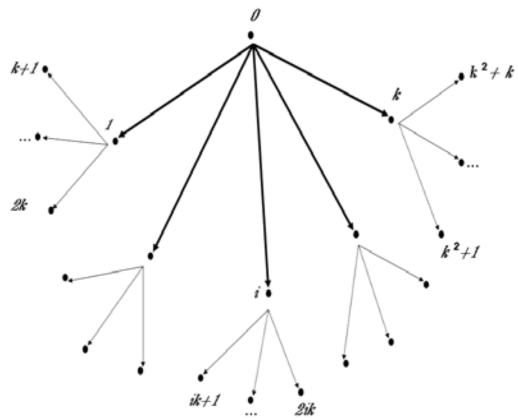


Figure 2: **Information transmission in tree network.** The set of linked nodes represents social interaction among insiders. Arrows indicate the direction of information transmission. For all $i = 1, ..., k$, "node" insider $i$ has private signal and additional information from the "root" insider 0. For all $i = 1, ..., k$ and $j = 1, ..., k$, "leave" insider $ik + j$ has private signal and additional signal from the "node" insider $i$.

The difference of information set leads to different trading strategies among root, nodes and leaves.

**Theorem 3** *With information transmission in financial tree network, there exists a unique linear equilibrium given by (subscript $T$ denotes star network)*

$$X_{T_0}(\tilde{s}_0) = \alpha_T \tilde{s}_0 \tag{5.1}$$

$$X_{Ti}(\tilde{s}_i, \tilde{s}_0) = \beta_T \tilde{s}_i + \gamma_T \tilde{s}_0 \text{ for } i = 1, \cdots, k \tag{5.2}$$

$$X_{T,ik+j}(\tilde{s}_{ik+j}, \tilde{s}_i) = \delta_T \tilde{s}_{ik+j} + \phi_T \tilde{s}_i \text{ for } i = 1, \cdots, k \text{ and } j = 1, \cdots, k \tag{5.3}$$

$$P_T(\tilde{\omega}) = \lambda_T \left( \sum_{i=0}^{n-1} \tilde{x}_{Ti} + \tilde{u} \right) \tag{5.4}$$

*where trading intensity parameters $\alpha_T$, $\beta_T$, $\gamma_T$, $\delta_T$, $\phi_T$ and liquidity parameters $\lambda_T$ are* *given by*

$$\phi_T = \frac{(k^2+2k+6)\sigma_v^6+(4k-2)\sigma_v^4\sigma_\varepsilon^2+4\sigma_v^2\sigma_\varepsilon^4}{\begin{array}{c}[(k^2+2k+6)(k^2+2k+2)\sigma_v^6 \\ +(3k^4+16k^3+38k^2+90k+74)\sigma_v^4\sigma_\varepsilon^2 \\ +(k^4+6k^3+26k^2+54k+48)\sigma_v^2\sigma_\varepsilon^4+(4k^2+12k+8)\sigma_\varepsilon^6]\lambda\end{array}} = \frac{E}{\lambda_T} \tag{5.5}$$

$$\delta_T = \frac{\sigma_v^2}{\lambda}+[(3k+3)\sigma_v^2+(k+2)\sigma_\varepsilon^2]\frac{E}{\lambda} = \frac{D}{\lambda_T} \tag{5.6}$$

$$\beta_T = 2\frac{D}{\lambda}-(k+1)\frac{E}{\lambda} = \frac{B}{\lambda_T} \tag{5.7}$$

$$\alpha_T = \left\{\begin{array}{c}\frac{(k+1)\sigma_v^2}{(\sigma_v^2+\sigma_\varepsilon^2)(k+2)\lambda}-[\frac{(k^2+3k+6)\sigma_v^2+4k\sigma_\varepsilon^2}{(\sigma_v^2+\sigma_\varepsilon^2)(k+2)}]\frac{D}{\lambda_T} \\ +[\frac{(2k^2+3k)\sigma_v^2+(k^2+2k)\sigma_\varepsilon^2}{(\sigma_v^2+\sigma_\varepsilon^2)(k+2)}]\frac{E}{\lambda_T}\end{array}\right\} = \frac{A}{\lambda_T} \tag{5.8}$$

$$\gamma_T = \frac{1}{k+1}[4\frac{D}{\lambda_T}-(k+2)\frac{E}{\lambda_T}-\frac{A}{\lambda_T}] = \frac{C}{\lambda_T} \tag{5.9}$$

$$\lambda_T = \frac{1}{\sigma_u}\left[\begin{array}{c}\left(\begin{array}{c}\left[A+(B+C)\,k+(D+E)k^2\right] \\ -\left[A+(B+C)\,k+(D+E)k^2\right]^2\end{array}\right)\sigma_v^2 \\ -\left[(A+Ck)^2+k(B+Ek)^2+k^2D^2\right]\sigma_\varepsilon^2\end{array}\right]^{1/2} \tag{5.10}$$

*Here the algorithm is like the following: Given the market and information parameters summarized by $(k,\sigma_v^2,\sigma_\varepsilon^2,\sigma_u^2)$. $E$ is computed from 5.5. In turn $D$, $B$, $A$ and $C$ are derived from 5.6 - 5.9 Finally $\lambda_T$ is obtained from 5.10. Although the expression is quite involved, the closed form solution exists. We put the detail derivation in appendix. Also easy to check the second order condition $\lambda_\star > 0$ is satisfied.*

Compare the parameters in the above Theorem, we have the following proposition.

**Proposition 3** *For all node insiders, trading intensity $\beta_T$ is strictly larger than $\gamma_T$. For all leave insiders, trading intensity $\delta_T$ is strictly larger than $\phi_T$.*

This result might be surprising at first sight since it is different from the result in the circle network. An immediate investigation shows that the logic in circle network cannot be applied here. For Both node and leave insiders, although the received information is of the same precision as that of her private signal, she still has the incentives to count less on the received signal because of the special network structure. Every signal receiver clearly realizes that the received information is also utilized by all other insiders, while her private signal is unique to herself and gives her some monopolistic power. An optimal strategy therefore requires the receiver to give priority to her own private signal. The implication derived here is insiders might have different trading styles in terms of attitudes towards received and private signal in different social network.

Besides of trading intensity, we are more concerned about the influence of private signal on asset price because the volatile price change in financial markets is usually not accompanied with fundamental changes. Apparently, in circle network the influence of each insider's private signal on equilibrium price is identical because everyone interacts with each other symmetrically[6] and the signal is identically and independently distributed. In tree network information transmits in an asymmetric way, this property is altered significantly

**Proposition 4** *The root insider's private signal has a greater influence on equilibrium price than that of any node insider. And the node insider's private signal has a greater influence on equilibrium price than that of any leave insider. The influence ratio of root to node and node to leave is upper bounded by* 2

The first part of this result is consistent with Ozsoylev (2003) whose model, à la Hellwig (1980), shows that the weight of each agent's signal is larger than that of her successor's signal in the price. While the second part contrasts sharply to the result in Ozsoylev (2003). in which they conclude that the weight of an agent's signal is proportional to the number of agents she precedes in the tree network. So, as the number of insiders in the network gets larger (i.e, as $k \to \infty$), the influence ratios of root to node and node to leave also go to infinity. In another word, the weight of each node or leave insider' signal becomes completely insignificant relative to the weight of root insider's signal when $k \to \infty$. In our paper, since the influence ratio is upper bounded by 2, no agent in the network has dominant power even in the extreme case that she has infinite successors. The reason for our bounded influence result lies in that every signal receiver, who can't observe the equilibrium price, takes into account the fact that her predecessor's private signal is utilized by all other successors, hence everyone rationally underreacts to received information. The predecessor's private signal is incorporated into price in a deliberately controlled fashion. While in Ozsoylev (2003) all successors trade competitively without the same strategic consideration; each exploits the received information up to her risk aversion and signal precision. As the number of successors goes to infinity, the predecessor's private signal is

[6] As mentioned above, circle is for descriptive convenience. Symmetric social interaction only requires that each insider receives information from, and transmits information to, the same number of insiders.

absorbed into the equilibrium price without bound[7]. As Ozsoylev (2003) and our paper point our, trading in network provides a possible explanation for the large price swing, like bubble or crash, in securities markets without prior significant change in fundamentals. It seems that our bounded influence result is more empirically grounded.

This proposition also draws some analog between our tree network model and information cascade models as mentioned earlier. With common market and information structures, the few predecessors differ from the successors in action timing, while the root insider differs from the node and leave in location, the former always has greater or even infinite influence on the latter's decision which eventually determines the public action outcome or public price.

As we pointed out earlier, the expression of the linear equilibrium in tree network is very complicated. This makes the analysis of market statistics quite challenging. A polar case of tree network, star network, becomes more appealing for the tractability reason. Moreover, stat network is also a desirable environment to examine the effect of difference in signal quality. In next section, we will focus on star network.

## 6 Trading in Star Network

In this section we consider a special case of tree financial network in which the social interaction is represented by a star. In this network, the first central informed trader is surrounded by other $n-1$ peripheral informed traders and figure 3 depicts the flow of information transmission. A simple motivation is that the central insider is an information "guru" with respect to others, so the peripheral insiders schmooze with the central eagerly or listen to her carefully. Another realistic example is that the central insider is the one who expresses his opinion on the stock bbs and the peripheral insiders are the readers.

We still assume away the strategic information transmission or information-based price manipulation. Peripheral insiders receive *truthful* private signal from the central.

We assume that the variance of the central insider's private signal is $var\left(\tilde{s}_1\right) = \sigma_v^2 + \kappa\sigma_\varepsilon^2$ where the precision parameter $\kappa$ satisfies $0 < \kappa \leq 1$. This assumption is mainly made for our first result about the relative influence of central insider's private signal on price. When comparing properties of insider trading in star network with those in other models, we set

[7] For this result, Ozsoylev (2003) also assumes that the variance of liquidity trading goes to infinity in order to separate the role of social interaction from the role played by price. In our model the level of noise trading does not matter for this result.

$\kappa$ equal 1 for consistency.

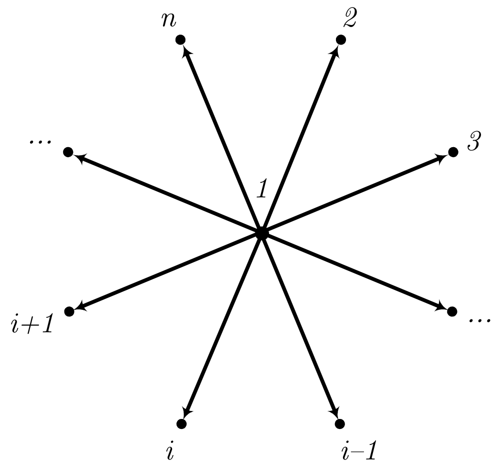


Figure 3: **Information transmission in star network.** The set of linked nodes represents social interaction among insiders. Arrows indicate the direction of information transmission. For all $i = 2, ..., n$, peripheral insider $i$ has private signal and additional information about the fundamental value of an asset from the central insider 1.

**Definition 4** *A Nash Equilibrium in Kyle's market with information transmission in financial star network represented above is similarly defined as Definition 1 except that (1) $\tilde{I}_1 = (\tilde{s}_1)$ and $\tilde{I}_i = (\tilde{s}_i, \tilde{s}_1)$ for $i = 2, \cdots, n$, (2) first informed trader has signal with variance $\sigma_v^2 + \kappa\sigma_\varepsilon^2$ where $0 < \kappa \leq 1$.*

The difference on information set leads to different trading strategies between central insider and the peripherals.

**Theorem 4** *With information transmission in financial star network, there exists a unique linear equilibrium given by (subscript $\star$ denotes star network)*

$$X_{\star 1}(\tilde{s}_1) = \alpha_\star \tilde{s}_1 \tag{6.1}$$

$$X_{\star i}\left(\tilde{s}_i, \tilde{\delta}_i\right) = \beta_\star \tilde{s}_i + \gamma_\star \tilde{s}_1 \text{ for } i = 2, \cdots, n \tag{6.2}$$

$$P_\star(\tilde{\omega}) = \lambda_\star \left(\sum_{i=1}^{n} \tilde{x}_{\star i} + \tilde{u}\right) \tag{6.3}$$

*where trading intensity parameters $a_\star$, $\beta_\star$ and $\gamma_\star$, and liquidity parameters $\lambda_\star$ are given by*

$$\alpha_\star = \frac{1}{\lambda_\star}\frac{\sigma_v^2}{(1+n)\left(\sigma_v^2+\kappa\sigma_\varepsilon^2\right)} \triangleq \frac{F}{\lambda_\star} \tag{6.4}$$

$$\beta_\star = \frac{1}{\lambda_\star}\frac{k\sigma_v^2}{(2+\kappa n)\,\sigma_v^2+2\kappa\sigma_\varepsilon^2} \triangleq \frac{G}{\lambda_\star} \tag{6.5}$$

$$\gamma_\star = \frac{1}{\lambda_\star}\frac{\sigma_v^2\left[(2-\kappa)\,\sigma_v^2+2\kappa\sigma_\varepsilon^2\right]}{(1+n)\left(\sigma_v^2+\kappa\sigma_\varepsilon^2\right)\left[(2+\kappa n)\,\sigma_v^2+2\kappa\sigma_\varepsilon^2\right]} \triangleq \frac{H}{\lambda_\star} \tag{6.6}$$

$$\lambda_\star = \frac{1}{\sigma_u}\left[\begin{array}{c}\left([F+(G+H)(n-1)]-[F+(G+H)(n-1)]^2\right)\sigma_v^2 \\ -\left(\kappa\,[F+H\,(n-1)]^2+G^2\,(n-1)\right)\sigma_\varepsilon^2\end{array}\right]^{1/2} \tag{6.7}$$

*and the second order condition $\lambda_\star > 0$ is satisfied.*

Again, we first solve $\lambda_\star$ and obtain $\alpha_\star$, $\beta_\star$, and $\gamma_\star$. The closed-form solution exists but it is still quite involved. The analytical and numerical methods are employed for the following propositions. $\sigma_v^2$ is normalized to 1 without affecting propositions.

As a simple case of tree network, star structure surely inherits the all the propositions in tree network.

**Proposition 5** *For all peripheral insider, trading intensity $\beta_\star$ is strictly larger than $\gamma_\star$. The central insider's private signal has a greater influence on equilibrium price than that of any other peripheral insider. The influence approaches* 2 *monotonically when $\kappa = 1$ and $n$ goes to infinity, and the influence approaches infinity monotonically when $\kappa$ goes to zero.*

The first and second parts of this proposition are exactly the same as those in last section. The third part is a result about the information quality. When central insider's private signal is extremely precise, each peripheral upgrades her trading intensity of received information over that of her private signal. In the limit, the central's signal can impact the equilibrium price unbounded.

In the following discussion, we restrict $\kappa = 1$ in order to contrast the difference among insider trading in star network and in benchmark. All of the following propositions hold regardless of the market and information structures summarized by parameters $\left(n, \sigma_v^2, \sigma_\varepsilon^2, \sigma_u^2\right)$

**Proposition 6** *Expected volume, price volatility and price efficiency in star network are higher than those in benchmark.*

# 7 Insider Trading in Complete Network

In this section, another interesting network which we call complete network is considered. The network is constructed like the following: every agent in the network learns from all other agents in the network, and every agent in the network is an information source of each agent in the same network. Formally, agent $i's$ information set $\tilde{I}_i = (\tilde{s}_i, \tilde{s}_0, \tilde{s}_1, \cdots \tilde{s}_{i-1}, \tilde{s}_{i+1}, \cdots, \tilde{s}_n)$ for $i = 1, \cdots, n$. In real stock market, there are many stock discussion groups formed by investors. The crucial element of information transmission captured by this complete network is that: in those discussion groups, people mostly learn from those in there network and rarely from others.

Still we keep the same assumption as in benchmark. Each insider $i$ possess a private signal $\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i$, where $\tilde{v}$ is normally distributed $\boldsymbol{N}\left(0, \sigma_v^2\right)$ and $\tilde{\varepsilon}_i$ is normally and identically distributed $\boldsymbol{N}\left(0, \sigma_\varepsilon^2\right)$ for $i = 1, \cdots, n$.

**Definition 5** *A Nash Equilibrium in Kyle's market with information transmission in financial complete network represented above is similarly defined as Definition 1 except that* $\tilde{I}_i = (\tilde{s}_i, \tilde{s}_0, \tilde{s}_1, \cdots \tilde{s}_{i-1}, \tilde{s}_{i+1}, \cdots, \tilde{s}_n)$ *for* $i = 1, \cdots, n$.

**Theorem 5** *With information transmission in financial complete network, there exists a unique linear equilibrium given by (subscript A denotes complete network)*

$$X_{Ai}\left(\tilde{s}_i, \tilde{s}_0, \tilde{s}_1, \cdots \tilde{s}_{i-1}, \tilde{s}_{i+1}, \cdots, \tilde{s}_n\right) = \sum_{i=1}^{n} \beta_{Ai} \tilde{s}_i \tag{7.1}$$

$$P_A\left(\tilde{\omega}\right) = \lambda_A \left(\sum_{i=1}^{n} \tilde{x}_i + \tilde{u}\right) \tag{7.2}$$

*where trading intensity parameters* $\beta$ *and liquidity parameters* $\lambda$ *are given by*

$$\beta_{Ai} = \sqrt{\frac{\sigma_u^2}{n^3\sigma_v^2 + n^2\sigma_\varepsilon^2}} \tag{7.3}$$

$$\lambda_A = \frac{n\sigma_v^2}{(n+1)\sigma_u\sqrt{n\sigma_v^2 + \sigma_\varepsilon^2}} \tag{7.4}$$

***Proposition 7*** *Expected trading volume and price volatility are higher in all-connected* *network relative to benchmark.*

$$\boldsymbol{E}\left(\widetilde{Vol}_{\circ}\right) = \frac{\sigma_u}{\sqrt{2\pi}}(\sqrt{n} + 1 + \sqrt{n+1}) > \boldsymbol{E}\left(\widetilde{Vol}\right)$$

$$\boldsymbol{var}\,(\tilde{p}_A) = \frac{n^2\sigma_v^4}{(n+1)(n\sigma_v^2 + \sigma_\varepsilon^2)} > \boldsymbol{var}\,(\tilde{p})$$

# 8 Conclusion

In this paper we generalize the strategical noisy rational expectation model by incorporating direct and truthful information transmission in financial networks. In particular, we introduce four major social interaction structures, circle, tree, star and complete networks, all of which accommodate the main features of information transmission in virtually all financial markets. Our model falls short of more complicated network structures such as complete network graph[8], nonetheless, the tractability of these four networks shed light on individual trading behavior and market trading patterns in both symmetrical and asymmetrical, one-way and two-way information transmission. We showed in this paper, individual trading behavior and influential power on price highly depend on network structure. In symmetrical circle and all-connected networks, insiders treat private and received information equally. In contrast, in asymmetrical tree and star networks, insider relies more on her private signal than the received signal which is also utilized by other insiders. More interestingly, we find in tree and star networks, higher hierarchy insiders have larger but not dominant influence on market price than there successors. Also we highlight that despite the distinction on individual trading behavior and influential power, the market trading patterns as summarized by volume, price volatility and price efficiency, are all similarly higher than those in model without network. Our model accords with empirical findings better than conventional models. The networks model also exhibit flexibility in that it can be extended under more general assumptions about market and information structures. For instance, we can study the effects of insider and market maker's risk aversion, heterogeneous prior belief, and so on.

We point our several fruitful directions of further research. First, understanding the role of strategic or distorted information transmission within our model is worth exploring, the incentive and reputation considerations will make this topic challenging. Secondly, in

[8] A graph is complete if two nodes are linked in at least one direction

our model, like previous studies, networks represented by different graphs are exogenously given and there is no formation and evolution of their structures. The efficiency and stability of networks are under question[9], while their importance should not be ignored. Bala and Goyal (2000), Bushman and Indjejikian (1995), Fishman and Hagerty (1995), Shin and Singh (2003) and Cao (2006) provide some theoretical foundation of network formation and information disclosure. Third, Extension to multiple periods will facilitate the analysis of networks' dynamic effects. We leave these topics for further research.

# 9 Appendix: Mathematical Preliminaries

## 9.1 Projection Theorem

Let $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ be two normally distributed random vectors and

$$\begin{pmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \end{pmatrix} \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \text{ with } \boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix} \text{ and } \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix},$$

then $\boldsymbol{X}_1$ conditional on $\boldsymbol{X}_2$ has a normal distribution such that

$$\boldsymbol{X}_1 | \boldsymbol{X}_2 \sim \boldsymbol{N}\left(\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\left(\boldsymbol{X}_2 - \boldsymbol{\mu}_2\right), \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}\right)$$

## 9.2 Statistical Formula

Let $Y$ be a normally distributed random variable and $Y \sim \boldsymbol{N}(0, \Sigma)$, then $\boldsymbol{E}|Y| = \sqrt{2\Sigma/\pi}$

[9] Jackson (2000) provides a detailed discussion of network efficiency and stability. Here we adopt the definition by Bala and Goyal (2000).

A network is efficient if the sum of agents' payoff is the highest among all available networks.

A network is stable if the following two conditions are satisfied: (1) The static network is formed in a Nash manner; (2) For any number of agents and starting from any initial network, the dynamic process converges to the static Nash network.

# 10 Appendix: Proofs

## 10.1 Proofs of Section 3

**Proof of Theorem 1.** Insider $i$, taking (3.3) with subscript $j$, for $j \neq i$, and (3.4) as given, solves the following problem:

$$\max_{x_i} \boldsymbol{E}\left[ x_i \left( \tilde{v} - \lambda \left( \sum_{j \neq i} \beta \tilde{s}_j + x_i + \tilde{u} \right) \right) \middle| \tilde{s}_i = s_i \right]$$

which is equivalent to (3.1) and can be rewritten as

$$\max_{x_i} x_i \left[ \left( \frac{\sigma_v^2 s_i}{\sigma_v^2 + \sigma_\varepsilon^2} \right) (1 - \lambda (n-1) \beta) - \lambda x_i \right]$$

The solution to this problem is given by

$$x_i^* = \frac{(1 - \lambda (n-1) \beta) \sigma_v^2}{2\lambda (\sigma_v^2 + \sigma_\varepsilon^2)} s_i$$

A Nash equilibrium among insiders is found by solving the following equation:

$$\beta = \frac{(1 - \lambda (n-1) \beta) \sigma_v^2}{2\lambda (\sigma_v^2 + \sigma_\varepsilon^2)} \tag{10.1}$$

The pricing rule set by market maker must satisfy (3.2). Taking (3.3) as given,

$$\boldsymbol{E}(\tilde{v} | \tilde{\omega} = \omega) = \frac{\boldsymbol{cov}(\tilde{v}, \tilde{\omega})}{\boldsymbol{var}(\tilde{\omega})} \omega$$

therefore (3.4) satisfies (3.2) if $\lambda$ satisfies

$$\begin{aligned} \lambda &= \frac{\boldsymbol{cov}(\tilde{v}, \tilde{\omega})}{\boldsymbol{var}(\tilde{\omega})} = \frac{\boldsymbol{cov}(\tilde{v}, \sum \beta (\tilde{v} + \tilde{\varepsilon}_i) + \tilde{u})}{\boldsymbol{var}(\sum \beta (\tilde{v} + \tilde{\varepsilon}_i) + \tilde{u})} \\ &= \frac{n \beta \sigma_v^2}{n^2 \beta^2 \sigma_v^2 + n \beta^2 \sigma_\varepsilon^2 + \sigma_u^2} \end{aligned} \tag{10.2}$$

Solving (10.1) and (10.2) yields $\beta$ and $\lambda$ as given in (3.5) and (3.6) on page 9. The second order condition $\lambda > 0$ is satisfied. ■

## 10.2 Proofs of Section 4

**Proof of Theorem 2.** For notational clarity, we suppress the subscripts $\circ$ in $\beta_\circ$, $\gamma_\circ$, $\lambda_\circ$, and $x_{\circ i}$ in the following proof.

Insider $i$, taking (4.1) with subscript $j$, for $j \neq i$, and (4.2) as given, solves the following problem:

$$\max_{x_i} \boldsymbol{E}\left( x_i \left[ \tilde{v} - \lambda \left( \sum_{j \neq i} (\beta \tilde{s}_j + \gamma \tilde{s}_{j+1}) + x_i + \tilde{u} \right) \right] \middle| \tilde{s}_i = s_i, \tilde{s}_{i+1} = s_{i+1} \right)$$

which can be rewritten as

$$\max_{x_i} x_i \left( \boldsymbol{E}\left[ \tilde{v} | \, s_i, s_{i+1} \right] - \lambda x_i - \lambda\beta \sum_{j \neq i} \boldsymbol{E}\left[ \tilde{s}_j | \, s_i, s_{i+1} \right] - \lambda\gamma \sum_{j \neq i} \boldsymbol{E}\left[ \tilde{s}_{j+1} | \, s_i, s_{i+1} \right] \right)$$

The solution to this problem is given by

$$\begin{aligned}
x_i^* &= \frac{1}{2\lambda} \boldsymbol{E}\left[ \tilde{v} | \, s_i, s_{i+1} \right] - \frac{1}{2} \left( \beta \sum_{j \neq i} \boldsymbol{E}\left[ \tilde{s}_j | \, s_i, s_{i+1} \right] + \gamma \sum_{j \neq i} \boldsymbol{E}\left[ \tilde{s}_{j+1} | \, s_i, s_{i+1} \right] \right) \\
&= \frac{1}{2\lambda} \frac{\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} (s_i + s_{i+1}) - \frac{1}{2} \left( \begin{array}{c} \beta[s_{i+1} + \frac{(n-2)\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2}(s_i + s_{i+1})] \\ +\gamma[s_i + \frac{(n-2)\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2}(s_i + s_{i+1})] \end{array} \right) \\
&= \left\{ \left[ \frac{1}{2\lambda} - \frac{(n-2)(\beta+\gamma)}{2} \right] \frac{\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} - \frac{\gamma}{2} \right\} s_i \\
&\quad + \left\{ \left[ \frac{1}{2\lambda} - \frac{(n-2)(\beta+\gamma)}{2} \right] \frac{\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} - \frac{\beta}{2} \right\} s_{i+1}
\end{aligned}$$

■

Nash equilibrium indicates,

$$\begin{aligned}
\beta &= \left[ \frac{1}{2\lambda} - \frac{(n-2)(\beta+\gamma)}{2} \right] \frac{\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} - \frac{\gamma}{2} \\
\gamma &= \left[ \frac{1}{2\lambda} - \frac{(n-2)(\beta+\gamma)}{2} \right] \frac{\sigma_v^2}{2\sigma_v^2 + \sigma_\varepsilon^2} - \frac{\beta}{2}
\end{aligned}$$

both of which yield

$$\beta = \gamma = \frac{1}{2\lambda \left[ (n-2) + \frac{6\sigma_v^2 + 3\sigma_\varepsilon^2}{2\sigma_v^2} \right]} \tag{10.3}$$

Market maker sets the semi-strong efficient pricing rule in the familiar manner, yielding

$$\lambda = \frac{\boldsymbol{cov}\left(\sum_{i=1}^n \tilde{x}_i + \tilde{u}, \tilde{v}\right)}{\boldsymbol{var}\left(\sum_{i=1}^n \tilde{x}_i + \tilde{u}\right)} = \frac{n(\beta+\gamma)\sigma_v^2}{n^2(\beta+\gamma)^2\sigma_v^2 + n(\beta+\gamma)^2\sigma_\varepsilon^2 + \sigma_u^2}$$

Substituting in for $\beta$ and $\gamma$ as given in (10.3) yields 4.3, 4.4 and (4.5).

The second order condition $\lambda > 0$ is shown to be satisfied.

**Proof of Proposition 1.** Expected trading volume in circle network is

$$\begin{aligned}
\boldsymbol{E}\left(\widetilde{Vol_\circ}\right) &= \frac{1}{\sqrt{2\pi}}\left\{ n\sqrt{(\beta_\circ+\gamma_\circ)^2\sigma_v^2 + \left(\beta_\circ^2+\gamma_\circ^2\right)\sigma_\varepsilon^2} + \sigma_u + \sqrt{(\beta_\circ+\gamma_\circ)^2(n^2\sigma_v^2 + n\sigma_\varepsilon^2) + \sigma_u^2} \right\} \\
&= \frac{\sigma_u}{\sqrt{2\pi}}\left[ \sqrt{n} + 1 + \sqrt{\frac{(2n+2)\sigma_v^2 + 3\sigma_\varepsilon^2}{2\sigma_v^2 + \sigma_\varepsilon^2}} \right]
\end{aligned}$$

To compare $\boldsymbol{E}\left(\widetilde{Vol_\circ}\right)$ and $\boldsymbol{E}\left(\widetilde{Vol}\right)$, we have

$$\begin{aligned}
\boldsymbol{E}\left(\widetilde{Vol_\circ}\right) - \boldsymbol{E}\left(\widetilde{Vol}\right) &= \frac{\sigma_u}{\sqrt{2\pi}}\left[ \sqrt{n} + 1 + \sqrt{\frac{(2n+2)\sigma_v^2 + 3\sigma_\varepsilon^2}{2\sigma_v^2 + \sigma_\varepsilon^2}} \right] \\
&\quad - \frac{\sigma_u}{\sqrt{2\pi}}\left( 1 + \sqrt{n} + \sqrt{\frac{(1+n)\sigma_v^2 + 2\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}} \right) \\
&= \frac{\sigma_u}{\sqrt{2\pi}}\left[ \sqrt{\frac{(2n+2)\sigma_v^2 + 3\sigma_\varepsilon^2}{2\sigma_v^2 + \sigma_\varepsilon^2}} - \sqrt{\frac{(1+n)\sigma_v^2 + 2\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}} \right] \\
&> 0
\end{aligned}$$

■

**Proof of Proposition 2.** Price volatility in circle network is.

$$\begin{aligned}
\boldsymbol{var}\left(\tilde{p}_\circ\right) &= \lambda_\circ^2\left[n^2\left(\beta_\circ+\gamma_\circ\right)^2\sigma_v^2+n\left(\beta_\circ+\gamma_\circ\right)^2\sigma_\varepsilon^2+\sigma_u^2\right] && (10.4)\\
&= \frac{n\sigma_v^4}{(1+n)\sigma_v^2+\frac{3}{2}\sigma_\varepsilon^2} && (10.5)\\
&> \boldsymbol{var}\left(\tilde{p}\right)=\frac{n\sigma_v^4}{(1+n)\,\sigma_v^2+2\sigma_\varepsilon^2} && (10.6)
\end{aligned}$$

Price efficiency in circle network is.

$$\begin{aligned}
\left[\boldsymbol{var}\left(\tilde{v}|\tilde{p}_\circ\right)\right]^{-1} &= \left[\sigma_v^2-n\left(\beta_\circ+\gamma_\circ\right)\lambda_\circ\sigma_v^2\right]^{-1}=\frac{(1+n)\,\sigma_v^2+\frac{3}{2}\sigma_\varepsilon^2}{\sigma_v^4+\frac{3}{2}\sigma_v^2\sigma_\varepsilon^2}\\
&> \left[\boldsymbol{var}\left(\tilde{v}|\tilde{p}\right)\right]^{-1}=\frac{(1+n)\,\sigma_v^2+2\sigma_\varepsilon^2}{\sigma_v^4+2\sigma_v^2\sigma_\varepsilon^2}
\end{aligned}$$

■

### 10.3 Proofs of Section 5

**Proof of theorem 3.** For notational clarity, we suppress the subscripts $T$ in $\alpha_T$, $\beta_T$, $\gamma_T$, $\delta_T$, $\phi_T$, $\lambda_T$, and $x_{Ti}$ in the following proof

"Root" Insider 0, taking (5.2),5.3 for $i\neq 0$, and (5.4) as given, solves the following problem:

$$\max_{x_0}\boldsymbol{E}\left(x_0\left[\tilde{v}-\lambda\left(x_0+\sum_{i=1}^{k}\left(\beta\tilde{s}_i+\gamma\tilde{s}_0\right)+\sum_{i=1}^{k}\left(\sum_{j=1}^{k}\delta\tilde{s}_{ik+j}+\phi\tilde{s}_i\right)+\tilde{u}\right)\right]\middle|\,\tilde{s}_0=s_0\right)$$

which can be rewritten as

$$\max_{x_0}\left\{\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}\left[1-\lambda\beta k-\lambda(\delta+\phi)k^2\right]-\lambda\gamma k\right\}s_0x_0-\lambda x_0^2 \tag{10.7}$$

The solution to this problem is given by

$$x_0=\frac{1}{2\lambda}\left\{\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}\left[1-\lambda\beta k-\lambda(\delta+\phi)k^2\right]-\lambda\gamma k\right\}s_0$$

The second order condition is $\lambda>0$.

"Nodes" insider $i \in \{1, \cdots, k\}$, taking (5.1), (5.3) and (5.4) as given, solves:

$$\max_{x_i} \boldsymbol{E}\left( x_i \left[ \tilde{v} - \lambda \left( \begin{array}{c} \alpha\tilde{s}_0 + x_i + \sum_{i\in\{1,\cdots,k\},j\neq i} (\beta\tilde{s}_j + \gamma\tilde{s}_0) \\ + \sum_{t=1}^{k} \left( \sum_{j=1}^{k} \delta\tilde{s}_{tk+j} + \phi\tilde{s}_t \right) + \tilde{u} \end{array} \right) \right] \middle| \tilde{s}_i = s_i, \tilde{s}_0 = s_0 \right)$$

which can be rewritten as

$$\max_{x_i} x_i \left\{ \begin{array}{c} \frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2} \left[1 - \lambda\beta\,(k-1) - \lambda\delta k^2 - \lambda\phi(k^2-k)\right](s_i + s_0) \\ -\lambda\left\{[\alpha + \gamma\,(k-1)]\, s_0 + \phi k \boldsymbol{s}_i\right\} \end{array} \right\} - \lambda x_i^2$$

Because $\boldsymbol{E}\left(\tilde{v}|s_i, s_0\right) = \boldsymbol{E}\left(\tilde{s}_j|s_i, s_0\right) = \frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2}\left(s_i + s_0\right)$. The solution to this problem is given by

$$x_i = \frac{1}{2\lambda}\left\{ \frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2} \left[ \begin{array}{c} 1 - \lambda\beta\,(k-1) \\ -\lambda\delta k^2 - \lambda\phi(k^2-k) \end{array} \right](s_i + s_0) - \lambda\left\{[\alpha + \gamma\,(k-1)]\, s_0 + \phi k \boldsymbol{s}_i\right\} \right\}$$

The second order condition is $\lambda > 0$

"Leaves" insider $ik+j$, here $i \in \{1, \cdots, k\}$, $j \in \{1, \cdots, k\}$, taking (5.1), (5.2) and (5.4) as given, solves:

$$\max_{x_{ik+j}} \boldsymbol{E}\left( x_{ik+j} \left[ \tilde{v} - \lambda \left( \begin{array}{c} \alpha\tilde{s}_0 + \sum_{i=1}^{k} (\beta\tilde{s}_i + \gamma\tilde{s}_0) \\ + \sum_{i=1}^{k} \left( \sum_{j=1}^{k} \delta\tilde{s}_{ik+j} + \phi\tilde{s}_i \right) + \tilde{u} \end{array} \right) \right] \middle| \tilde{s}_{ik+j} = s_{ik+j}, \tilde{s}_i = s_i \right)$$

which can be rewritten as

$$\max_{x_{ik+j}} x_{ik+j} \left\{ \frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2} \left[ \begin{array}{c} 1 - \lambda\alpha - \lambda\beta\,(k-1) \\ -\lambda\delta(k^2-1) - \lambda\phi(k^2-k) \end{array} \right](s_{ik+j} + s_i) - \lambda\left[\beta + \phi(k-1)\right] s_i \right\} - \lambda x_{ik+j}^2$$

The solution to this problem is given by

$$x_{ik+j} = \frac{1}{2\lambda}\left\{ \frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2} \left[ \begin{array}{c} 1 - \lambda\alpha - \lambda\beta\,(k-1) \\ -\lambda\delta(k^2-1) - \lambda\phi(k^2-k) \end{array} \right](s_{ik+j} + s_i) - \lambda\left[\beta + \phi(k-1)\right] s_i \right\}$$

The second order condition is $\lambda > 0$

Imposing Nash equilibrium requirement yields

$$\alpha = \frac{1}{2\lambda}\left\{\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}\left[1-\lambda\beta k-\lambda(\delta+\phi)k^2\right]-\lambda\gamma k\right\} \tag{10.8}$$

$$\beta = \frac{1}{2\lambda}\left\{\frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2}\left[1-\lambda\beta\,(k-1)-\lambda\delta k^2-\lambda\phi(k^2-k)\right]-\lambda\phi k\right\} \tag{10.9}$$

$$\gamma = \frac{1}{2\lambda}\left\{\frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2}\left[1-\lambda\beta\,(k-1)-\lambda\delta k^2-\lambda\phi(k^2-k)\right]-\lambda\left[\alpha+\gamma\,(k-1)\right]\right\} \tag{10.10}$$

$$\delta = \frac{1}{2\lambda}\left\{\frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2}\left[1-\lambda\alpha-\lambda\beta\,(k-1)-\lambda\delta(k^2-1)-\lambda\phi(k^2-k)\right]\right\} \tag{10.11}$$

$$\phi = \frac{1}{2\lambda}\left\{\frac{\sigma_v^2}{2\sigma_v^2+\sigma_\varepsilon^2}\left[\begin{array}{c}1-\lambda\alpha-\lambda\beta\,(k-1)\\ -\lambda\delta(k^2-1)-\lambda\phi(k^2-k)\end{array}\right]-\lambda\left[\beta+\phi(k-1)\right]\right\} \tag{10.12}$$

From 10.8-10.12 We derive

$$\phi = \frac{(k^2+2k+6)\sigma_v^6+(4k-2)\sigma_v^4\sigma_\varepsilon^2+4\sigma_v^2\sigma_\varepsilon^4}{\begin{array}{c}[(k^2+2k+6)(k^2+2k+2)\sigma_v^6+(3k^4+16k^3+38k^2+90k+74)\sigma_v^4\sigma_\varepsilon^2\\ +(k^4+6k^3+26k^2+54k+48)\sigma_v^2\sigma_\varepsilon^4+(4k^2+12k+8)\sigma_\varepsilon^6]\lambda\end{array}} = \frac{E}{\lambda} \tag{10.13}$$

$$\delta = \frac{\sigma_v^2+[(3k+3)\sigma_v^2+(k+2)\sigma_\varepsilon^2]E}{\lambda[(k^2+2k+6)\sigma_v^2+4\sigma_\varepsilon^2]} = \frac{D}{\lambda} \tag{10.14}$$

$$\beta = 2\delta-(k+1)\phi = 2\frac{D}{\lambda}-(k+1)\frac{E}{\lambda} = \frac{B}{\lambda} \tag{10.15}$$

$$\alpha = \left\{\begin{array}{c}\frac{(k+1)\sigma_v^2}{(\sigma_v^2+\sigma_\varepsilon^2)(k+2)\lambda}-[\frac{(k^2+3k+6)\sigma_v^2+4k\sigma_\varepsilon^2}{(\sigma_v^2+\sigma_\varepsilon^2)(k+2)}]\delta\\ +[\frac{(2k^2+3k)\sigma_v^2+(k^2+2k)\sigma_\varepsilon^2}{(\sigma_v^2+\sigma_\varepsilon^2)(k+2)}]\phi\end{array}\right\} = \frac{A}{\lambda} \tag{10.16}$$

$$\gamma = \frac{1}{k+1}[4\delta-(k+2)\phi-a] = \frac{C}{\lambda} \tag{10.17}$$

Market maker takes (5.1) , (5.2) and (5.3) as given, sets $\lambda$ in the familiar manner,

$$\lambda = \frac{\left[\alpha+(\beta+\gamma)\,k+(\delta+\phi)k^2\right]\sigma_v^2}{\left[\alpha+(\beta+\gamma)\,k+(\delta+\phi)k^2\right]^2\sigma_v^2+\left[(\alpha+\gamma k)^2+k(\beta+\phi k)^2+k^2\delta^2\right]\sigma_\varepsilon^2+\sigma_u^2}$$

Substituting in for $\alpha$, $\beta$, $\gamma, \delta$ and $\phi$ as given in (10.13)-(10.17) yields .

$$\begin{aligned}
\lambda &= \frac{\left[\frac{A}{\lambda}+\left(\frac{B}{\lambda}+\frac{C}{\lambda}\right)k+(\frac{D}{\lambda}+\frac{E}{\lambda})k^2\right]\sigma_v^2}{\left[\frac{A}{\lambda}+\left(\frac{B}{\lambda}+\frac{C}{\lambda}\right)k+(\frac{D}{\lambda}+\frac{E}{\lambda})k^2\right]^2\sigma_v^2+\left[\left(\frac{A}{\lambda}+\frac{C}{\lambda}k\right)^2+k(\frac{B}{\lambda}+\frac{E}{\lambda}k)^2+k^2(\frac{D}{\lambda})^2\right]\sigma_\varepsilon^2+\sigma_u^2} \\
&= \frac{1}{\sigma_u}\sqrt{\begin{array}{c}\left[A+(B+C)k+(D+E)k^2\right]\sigma_v^2-\left[A+(B+C)k+(D+E)k^2\right]^2\sigma_v^2 \\ -\left[(A+Ck)^2+k(B+Ek)^2+k^2D^2\right]\sigma_\varepsilon^2\end{array}}
\end{aligned}$$

The second order condition $\lambda > 0$ is shown to be satisfied with numerical analysis.

**Proof of Proposition 3.** From 10.8 - 10.12, We derive

$$\delta-\phi = \frac{1}{2}\left[\beta+\phi(k-1)\right]>0 \tag{10.18}$$

$$\beta-\gamma = \frac{1}{2}[\alpha+\gamma(k-1)-\phi k]>0 \tag{10.19}$$

Equation 10.18 and 10.19 can be quickly verified after plug in the expression in 10.13-10.17.
■

**Proof of Proposition 4.** 5.4 shows

$$\begin{aligned}
P_T(\tilde{\omega}) &= \lambda_T\left(\sum_{i=0}^{n-1}\tilde{x}_{Ti}+\tilde{u}\right) \\
&= \lambda_T\left(\alpha\tilde{s}_0+\sum_{i=1}^{k}(\beta\tilde{s}_i+\gamma\tilde{s}_0)+\sum_{i=1}^{k}\left(\sum_{j=1}^{k}\delta\tilde{s}_{ik+j}+\phi\tilde{s}_i\right)+\tilde{u}\right) \\
&= \lambda_T\left((\alpha+k\gamma)\tilde{s}_0+\sum_{i=1}^{k}(\beta+k\phi)\tilde{s}_i+\sum_{i=1}^{k}\sum_{j=1}^{k}\delta\tilde{s}_{ik+j}+\tilde{u}\right)
\end{aligned}$$

Thus the influence ratio of "root" insider to "node" insider is

$$\frac{\alpha+k\gamma}{\beta+k\phi}=\frac{2\beta+k\phi-\gamma}{\beta+k\phi}=2-\frac{\gamma+k\phi}{\beta+k\phi}\in(1,2)$$

Here the first equality uses $\alpha = 2\beta + k\phi - (k+1)\gamma$ derived from 10.9 and 10.10 and the last relationship immediately follows from $\beta > \gamma$.

Similarly, the influence ratio of "node" insider to "leave" insider is

$$\frac{\beta+k\phi}{\delta}=\frac{2\delta-\phi}{\delta}=2-\frac{\phi}{\delta}\in(1,2)$$

Here the first equality uses $\beta = 2\delta - (k+1)\phi$ derived from 10.10 and 10.11 and the last relationship immediately follows from $\delta > \phi$. ■ ■

### 10.4 Proofs in Section 6

**Proof of Theorem 4.** For notational clarity, we suppress the subscripts $\star$ in $\alpha_\star$, $\beta_\star$, $\gamma_\star$, $\lambda_\star$, and $x_{\star i}$ in the following proof.

Central Insider 1, taking (6.2) for $i \neq 1$, and (6.3) as given, solves the following problem:

$$\max_{x_1} \boldsymbol{E}\left( x_1 \left[ \tilde{v} - \lambda \left( x_1 + \sum_{i=2}^{n} (\beta \tilde{s}_i + \gamma \tilde{s}_1) + \tilde{u} \right)\right] \middle| \tilde{s}_1 = s_1 \right)$$

which can be rewritten as

$$\max_{x_1} \frac{\sigma_v^2}{\sigma_v^2 + \kappa\sigma_\varepsilon^2} \left( [1 - \lambda\beta (n-1)] - \lambda\gamma (n-1) \right) s_1 x_1 - \lambda x_1^2 \tag{10.20}$$

because $E(\tilde{v}|s_1) = E(\tilde{s}_i|s_1) = \sigma_v^2 s_1 / \left(\sigma_v^2 + \kappa\sigma_\varepsilon^2\right)$, and $E(\tilde{s}_1|s_1) = s_1$. The second order condition is $\lambda > 0$.

Peripheral insider $i \neq 1$, taking (6.1), (6.2) with subscript $j > 1$ and $j \neq i$, and (6.3), solves:

$$\max_{x_i} \boldsymbol{E}\left( x_i \left[ \tilde{v} - \lambda \left( \alpha\tilde{s}_1 + x_i + \sum_{j>1, j\neq i} (\beta \tilde{s}_j + \gamma \tilde{s}_1) + \tilde{u} \right)\right] \middle| \tilde{s}_i = s_i, \tilde{s}_1 = s_1 \right)$$

which can be rewritten as

$$\max_{x_i} x_i \left[ \boldsymbol{E}(\tilde{v}|s_i, s_1) - \lambda \left( [\alpha + \gamma (n-2)] s_1 + \beta \sum_{j>1,\, j\neq i} \boldsymbol{E}(\tilde{s}_j|s_i, s_1) \right)\right] - \lambda x_i^2 \tag{10.21}$$

The second order condition is $\lambda > 0$ and it can be shown that

$$\boldsymbol{E}(\tilde{v}|s_i, s_1) = \boldsymbol{E}(\tilde{s}_j|s_i, s_1) = \frac{\sigma_v^2}{(1+\kappa)\sigma_v^2 + \kappa\sigma_\varepsilon^2} (\kappa s_i + s_1)$$

Solving (10.20) and (10.21) and imposing Nash equilibrium requirement yields

$$\alpha = \frac{1}{2\lambda}\left[\frac{\sigma_v^2}{\sigma_v^2+\kappa\sigma_\varepsilon^2}\left(1-\lambda\beta\left(n-1\right)\right)\right]-\frac{1}{2}\gamma\left(n-1\right) \tag{10.22}$$

$$\beta = \frac{1}{2\lambda}\frac{\kappa\sigma_v^2\left(1-\lambda\beta\left(n-2\right)\right)}{\left(1+\kappa\right)\sigma_v^2+\kappa\sigma_\varepsilon^2} \tag{10.23}$$

$$\gamma = \frac{1}{2\lambda}\frac{\sigma_v^2\left(1-\lambda\beta\left(n-2\right)\right)}{\left(1+\kappa\right)\sigma_v^2+\kappa\sigma_\varepsilon^2}-\frac{\alpha+\gamma\left(n-2\right)}{2} \tag{10.24}$$

(6.4)-(6.6) follow from (10.22)-(10.24) on page 21. We define

$$\alpha \triangleq \frac{F}{\lambda},\ \beta \triangleq \frac{G}{\lambda},\ \text{and}\ \gamma \triangleq \frac{H}{\lambda}$$

and note that $F$, $G$, and $H$ are independent of $\lambda$.

Market maker takes (6.1) and (6.2) as given, therefore $\tilde{\omega}=\alpha\tilde{s}_1+\sum_{i=2}^{n}\left(\beta\tilde{s}_i+\gamma\tilde{s}_1\right)+\tilde{u}$. Market maker sets $\lambda$ in the familiar manner,

$$\lambda=\frac{\left[\alpha+\left(\beta+\gamma\right)\left(n-1\right)\right]\sigma_v^2}{\left[\alpha+\left(\beta+\gamma\right)\left(n-1\right)\right]^2\sigma_v^2+\left[\kappa\left(\alpha+\gamma\left(n-1\right)\right)^2+\beta^2\left(n-1\right)\right]\sigma_\varepsilon^2+\sigma_u^2}$$

Substituting in for $\alpha$, $\beta$, and $\gamma$ as given in (6.4)-(6.6) yields (6.7).

The second order condition $\lambda>0$ is shown to be satisfied with numerical analysis. ■

**Proof of Proposition 5.** In equilibrium,

$$\tilde{p}_\star=\lambda_\star\left(\alpha_\star\tilde{s}_1+\sum_{i=2}^{n}\left(\beta_\star\tilde{s}_i+\gamma_\star\tilde{s}_1\right)+\tilde{u}\right)$$

We have

$$\frac{\partial\tilde{p}_\star/\partial\tilde{s}_1}{\partial\tilde{p}_\star/\partial\tilde{s}_i}=\frac{\alpha_\star+\left(n-1\right)\gamma_\star}{\beta_\star}=\frac{\left(2n+\kappa\right)\sigma_v^2+2n\kappa\sigma_\varepsilon^2}{\kappa\left(1+n\right)\left(\sigma_v^2+\kappa\sigma_\varepsilon^2\right)}>1$$

for any $n>1$ and $0<\kappa\leq 1$. Moreover,

$$\frac{\partial\left(\left[\alpha_\star+\left(n-1\right)\gamma_\star\right]/\beta_\star\right)}{\partial n} > 0 \text{ and } \lim_{n\to\infty}\frac{\alpha_\star+\left(n-1\right)\gamma_\star}{\beta_\star}=\frac{2}{\kappa}\text{ for any }\kappa$$

$$\frac{\partial\left(\left[\alpha_\star+\left(n-1\right)\gamma_\star\right]/\beta_\star\right)}{\partial\kappa} < 0 \text{ and } \lim_{\kappa\to 0}\frac{\alpha_\star+\left(n-1\right)\gamma_\star}{\beta_\star}=\infty\text{ for any }n$$

■

**Proof of Proposition 6.** Follow the definitions of expected volume, price volatility and price efficiency. We calculate them as following:

$$\boldsymbol{E}\left(\widetilde{Vol}_\star\right) = \frac{1}{\sqrt{2\pi}}\left(\alpha_\star\sqrt{\sigma_v^2+\sigma_\varepsilon^2} + (n-1)\sqrt{(\beta_\star+\gamma_\star)^2\sigma_v^2 + \left(\beta_\star^2+\gamma_\star^2\right)\sigma_\varepsilon^2} + \sigma_u\right)$$
$$+\frac{1}{\sqrt{2\pi}}\sqrt{\left[\alpha_\star+(\beta_\star+\gamma_\star)(n-1)\right]^2\sigma_v^2 + \left[(\alpha_\star+\gamma_\star(n-1))^2+\beta_\star^2(n-1)\right]\sigma_\varepsilon^2+\sigma_u^2}$$

For price volatility and price efficiency

$$\begin{pmatrix}\tilde{v}\\ \tilde{p}_\star\end{pmatrix} \sim N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma_v^2 & \lambda_\star\left[\alpha_\star+(\beta_\star+\gamma_\star)(n-1)\right]\sigma_v^2\\ \lambda_\star\left[\alpha_\star+(\beta_\star+\gamma_\star)(n-1)\right]\sigma_v^2 & \boldsymbol{var}(\tilde{p}_\star)\end{pmatrix}\right)$$

therefore

$$\boldsymbol{var}(\tilde{p}_\star) = \lambda_\star^2\left(\left[\alpha_\star+(\beta_\star+\gamma_\star)(n-1)\right]^2\sigma_v^2 + \left[(\alpha_\star+\gamma_\star(n-1))^2+\beta_\star^2(n-1)\right]\sigma_\varepsilon^2+\sigma_u^2\right)$$
$$\left[\boldsymbol{var}(\tilde{v}|\tilde{p}_\star)\right]^{-1} = \frac{\left[\alpha_\star+(\beta_\star+\gamma_\star)(n-1)\right]^2\sigma_v^2 + \left[(\alpha_\star+\gamma_\star(n-1))^2+\beta_\star^2(n-1)\right]\sigma_\varepsilon^2+\sigma_u^2}{\sigma_v^2\left(\left[(\alpha_\star+\gamma_\star(n-1))^2+\beta_\star^2(n-1)\right]\sigma_\varepsilon^2+\sigma_u^2\right)}$$

Comparing these statistics with those in benchmark by using numerical method, we get the proposition. We leave the details to readers ■

## 10.5 Proofs in Section 7

**Proof of Theorem 5.** For notational clarity, we suppress the subscripts $A$ in $\beta_A$, $\lambda_A$, and $x_{Ai}$ in the following proof. Insider $i$ solves the following problem:

$$\max_{x_i}\boldsymbol{E}\left(x_i\left[\tilde{v}-\lambda\left(\sum_{j\neq i}\sum_{i=1}^n\beta\tilde{s}_i + x_i + \tilde{u}\right)\right]\middle|\,\tilde{s}_1=s_1,\cdots,\tilde{s}_n=s_n\right)$$

which can be rewritten as

$$\max_{x_i} x_i\left(\boldsymbol{E}\left[\tilde{v}\right|s_1,\cdots,s_n] - \lambda x_i - \lambda\sum_{j\neq i}\sum_{i=1}^n\beta s_i\right)$$

The solution to this problem is given by

$$x_i = \frac{1}{2\lambda}\left(\frac{\sigma_v^2}{\sigma_\varepsilon^2 + n\sigma_v^2} + \lambda(n-1)\beta\right)\sum_{i=1}^{n} s_i$$

Nash equilibrium indicates,

$$\beta = \frac{1}{2\lambda}\left(\frac{\sigma_v^2}{\sigma_\varepsilon^2 + n\sigma_v^2} + \lambda(n-1)\beta\right)$$

Market maker sets the semi-strong efficient pricing rule in the familiar manner, yielding

$$\lambda = \frac{\boldsymbol{cov}\left(\sum_{i=1}^{n}\tilde{x}_i + \tilde{u}, \tilde{v}\right)}{\boldsymbol{var}\left(\sum_{i=1}^{n}\tilde{x}_i + \tilde{u}\right)} = \frac{\sigma_v^2 n^2 \beta}{n^4\beta^2\sigma_v^2 + n^3\beta^2\sigma_\varepsilon^2 + \sigma_u^2}$$

Both of which yield 7.3, and 7.4.

The second order condition $\lambda > 0$ is shown to be satisfied ■

**Proof of Proposition 7.** Expected trading volume in complete network is

$$\begin{aligned}\boldsymbol{E}\left(\widetilde{Vol}_A\right) &= \frac{1}{\sqrt{2\pi}}\left\{n\beta\sqrt{n\sigma_v^2 + n\sigma_\varepsilon^2} + \sigma_u + \sqrt{n^2\beta^2(n^2\sigma_v^2 + n\sigma_\varepsilon^2) + \sigma_u^2}\right\} \\ &= \frac{\sigma_u}{\sqrt{2\pi}}(\sqrt{n} + 1 + \sqrt{n+1}) > \boldsymbol{E}\left(\widetilde{Vol}\right)\end{aligned}$$

The price volatility in complete network is

$$\begin{aligned}\boldsymbol{var}\left(\tilde{p}_A\right) &= \lambda_A^2\beta_A^2 n^2(n^2\sigma_v^2 + n\sigma_\varepsilon^2) + \lambda_A^2\sigma_u^2 \\ &= \frac{n^2\sigma_v^4}{(n+1)(n\sigma_v^2 + \sigma_\varepsilon^2)} > \boldsymbol{var}\left(\tilde{p}\right)\end{aligned}$$

■

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