Exclusionary Pricing and Rebates in a Network Industry*

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Abstract

We consider an incumbent firm and an entrant, both supplying a network good, where the entrant has lower marginal cost of production than the incumbent. The good is sold to m identical small buyers and 1 large buyer, who will buy a firm's good only if the firm's network has reached a certain minimum size. The incumbent disposes of an installed base (above the threshold), while the entrant's network has size zero at the outset. In order to reach the minimum size, the entrant has to attract the large buyer plus at least one small buyer. If firms can only use uniform flat prices, the game has two equilibria, one entry equilibrium and one miscoordination equilibrium where the incumbent prevails. We show that if both firms can use uniform rebate contracts, the incumbent can prevent entry for a large range of parameter values where it could not have done so under flat prices.

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1 Introduction (Non-technical summary)

Rebates, i.e. discounts applicable where a customer exceeds a specified target for sales in a defined period, have been suspected by competition authorities of helping dominant firms to artificially foreclose business opportunities for their competitors (see Gyselen, 2003). However, the economic rationale underlying such practices is not yet well understood.

The purpose of our paper is to explore the exclusionary potential of rebate arrangements in the presence of network externalities. We consider an industry composed of an incumbent firm and an entrant, both supplying a network good, where the entrant has lower marginal cost of production than the incumbent.

The good is sold to $m + 1$ different buyers, m identical small buyers and 1 large buyer. Buyers' valuation for the good is increasing in the number of other buyers buying that same good as well (i.e. in the size of the network). The two networks are not compatible with each other, i.e. network externalities can only arise among customers of the same firm.

For buyers to be willing to pay a positive price for joining a firm's network, this network must reach a certain minimum size. We assume that the incumbent disposes of an installed base, and so its network has reached this minimum size already, while the entrant's network has size zero at the outset. In order to reach the minimum size, the entrant has to attract the large buyer plus at least one small buyer.

The timing of the game is as follows: First, both the incumbent and the entrant simultaneously announce their (binding) rebate schemes, each specifying a sales target, a unit price for sales below and above this target, and possible lump-sum payments (either as "rewards" from the firm to the buyer or as franchise fee from the buyer to the firm). The rebate scheme has to be uniform across buyers, i.e. we do not allow firms to specify individualized rebate schemes for each buyer.

Once these offers have become common knowledge, buyers decide which firm to patronize, and how much to buy from this firm, in particular whether to buy a quantity below or above the target.

If firms can only use uniform flat prices, then the game has two equilibria: entry equilibria, where the entrant undercuts the incumbent, and all buyers buy from the entrant; and miscoordination equilibria, where all firms buy from the incumbent, although the entrant makes a better offer to them. Either of the two equilibria can arise under all parameter values.

The central result of our paper is that rebates may allow the incumbent to break entry equilibria where the incumbent could not have done so under uniform flat prices. For a wide range of parameter values, only the miscoordination equilibrium survives when rebates can be used. Exclusion is more likely to be feasible if the efficiency gap between the two firms is not too wide, and if the buyers are sufficiently fragmented.

The reason is that rebate schemes, when applied to buyers who differ in size, will be a tool of (de facto) discrimination, even if the schemes as such are uniform. Now, the possibility of discriminating between buyers and redistributing rents between them should help the entrant challenge the incumbent just as much as it helps the incumbent defend its monopoly position.

However, the incumbent has an installed base that provides its network with the minimum size, so it can serve all buyers who want to buy from it, no matter how many (or how few) they are, while the entrant can only serve its buyers if it attracts at least large buyer plus one small buyer. Thus, if the large buyer decides to patronize the incumbent, then the small buyers have no other choice than to buy from the incumbent as well, and vice versa: If the small buyers prefer to buy from the incumbent, then the large buyer will be forced to do so as well, even if he prefers to buy from the entrant. Now, rebates allow the incumbent to play the two groups of buyers off against each other, which prevents them to coordinate on the more efficient supplier, and so entry will fail.

2 The setup

Consider an industry composed of two firms, the incumbent I, and an entrant E. The incumbent supplies a network good, and has an installed consumer base of size $\beta_I > 0$. I incurs marginal cost $c_I \in \left(0, \frac{1}{2}\right)$ for each unit it produces of the network good.

The entrant can supply a competing network good at marginal cost $c_E = 0$, so that $c_E < c_I$, i.e. the entrant is more efficient than the incumbent in supplying the good. E has not been active in the market so far, i.e. it has installed base $\beta_E = 0$, but it can start supplying the good any time; in particular, there is no need to sink any fixed costs of entry first.

The good can be sold to $m+1$ different buyers, indexed by $j = 1, \ldots, m+1$. There are m identical small buyers, and 1 large buyer. Goods acquired by one buyer cannot be resold to another buyer, nor can buyers make side payments of any kind to each other. Define firm i's network size s_i (where $i = I, E$) as

$$s_i = \beta_i + q_i^1 + \ldots + q_i^{m+1}$$

i.e. the firm's installed base and its total sales to all "new" buyers.

The large buyer's demand for firm i's network good at unit price $p_i^l \geq 0$ is given by

$$q_i^l\left(p_i^l\right) = \begin{cases} \max\left\{(1-K)\left(1-p_i^l\right), 0\right\} & \text{if } s_i \geq \bar{s} \\ 0 & \text{if } s_i < \bar{s} \end{cases} \tag{1}$$

while a typical small buyer's demand for firm i's network good at unit price $p_i^s \geq 0$ is

$$q_i^s\left(p_i^s\right) = \begin{cases} \max\left\{\frac{K}{m}\left(1-p_i^s\right), 0\right\} & \text{if } s_i \geq \bar{s} \\ 0 & \text{if } s_i < \bar{s} \end{cases} \tag{2}$$

The parameter $K \in (0,1)$ is an indicator of the relative importance of the small buyers in total market size: if all buyers buy at the same unit price (i.e. if

$p_i^l = p_i^s$), then $1-K$ measures the large buyer's market share, while K measures the market share of the group of small buyers. Assume that $1 - K > \frac{K}{m}$, so that if offered the same price, the large buyer's demand is always larger than a typical small buyer's demand.[1] Then, total potential market size is fixed at 1,

$$m\frac{K}{m} + (1-K) = 1.$$

Note that demand functions are identical across buyers up to the size factor ($1-K$ or $\frac{K}{m}$). Thus, in the absence of competitors, a monopolistic firm would charge the same price to all buyers, even if it could price-discriminate. We can conclude that if firms charge different unit prices to their buyers when facing a competitor, then this must be due to the competitive interaction between these firms, and has nothing to do with variations in the price-elasticity of demand across buyers.

Note also that the monopolist would set $p_i^* = \frac{1}{2}(1+c_i)$. Recall that $c_E = 0$, so that $p_E^* = \frac{1}{2}$; then, our assumption that $c_I < \frac{1}{2}$ implies that the entrant is never radically more efficient than the incumbent.

If firm i's network size is below the threshold level $\bar{s}$, no buyer (neither large nor small) would want to buy firm i's good. We assume that

$$\beta_I \geq \bar{s}$$

i.e. the incumbent has already reached the minimum size, while the entrant's installed base is $\beta_E = 0$. In order to operate successfully, the entrant will have to attract enough buyers to reach $\bar{s}$.[2]

Key Assumption: *In order to reach the minimum size, the entrant has to serve the large buyer plus at least one small buyer:*

$$\bar{s} > \max\ \{1-K, K\} \tag{3}$$

Thus, winning the large buyer's orders is indispensable for the entrant to operate successfully. However, neither demand of the large buyer alone, nor demand of all small buyers taken together, is sufficient for the entrant to reach the minimum size.[3]

We also assume that if the entrant gets to sell to all $m+1$ buyers at marginal cost, then it will reach the minimum size:

$$\bar{s} \leq mq_E^s(c_E) + q_E^l(c_E) = 1 \tag{4}$$

Note that inequality (4) together with $c_E < c_I$ imply that the social planner would want the entrant (and not the incumbent) to serve all buyers.

[1] Later, we will allow firm i's unit prices to vary with quantity, so that prices may differ across buyers of different size.

[2] Note that if the entrant manages to reach the minimum size $\bar{s}$, then consumers will consider I's and E's networks as being of homogenous quality, even if $s_I \neq s_E$.

[3] Note that only units which are actually consumed by a buyer count towards firm i's network size. Thus, we do not allow E to produce units and throw them away (or give them away for free to buyers who cannot consume them), in order to reach the minimum size.

Play occurs in the following sequence:

$t = 0$: The incumbent and the entrant simultaneously announce their price schemes, which will be binding in $t = 1$.[4]

$t = 1$: Each of the $m + 1$ buyers decides whether to patronize the incumbent or the entrant.

The rebate schemes offered by firms in $t = 0$ have to specify a sales target[5], a unit price for sales below and above this target, and possible lump-sum payments, either from the firms to the buyers ("rewards" for patronizing that firm), or from the buyers to the firms (franchise fees for being supplied).[6]

The rebate scheme has to be uniform across buyers, i.e. we do not allow firms to blatantly discriminate among their buyers by specifying individualized terms for each buyer. This assumption derives from EU case law on rebate practices, which considers individualized target rebates as equivalent to fidelity rebates, and hence per se illegal, while standardized target rebates are still lawful.[7]

Let us assume that offers are observable to everyone, e.g. because they have to be posted publicly. Then, when the buyers have to decide which firm to buy from, the firms' offers will be common knowledge. In $t = 1$, buyers decide which firm to buy from. For now, let us restrict attention to the case where a buyer can only buy from one of the two firms; later, we will also consider the case where a buyer may patronize both firms.

3 Characterizing equilibrium outcomes

As a benchmark case, let us consider the situation where firms can only use uniform flat prices (but no fixed payments or unit prices which vary with quantity). The following Proposition draws on the work of Bernheim and Whinston (1998) and Segal and Whinston (2000).

We introduce the tie-breaking rule that if buyers are indifferent between buying from I or from E, all buyers will buy from I. Then, our game has two types of equilibria:

Proposition 1 *(equilibria under uniform flat prices) If firms can only use uniform flat prices, the following two pure-strategy Nash equilibria exist under all parameter values (after eliminating all equilibria where firms play weakly dominated strategies):*

[4] In other words, firms can *commit* to make fixed payments at the end of $t = 1$ (buyers don't have to be concerned that firms renege on payments), so we exclude the possibility that firms just make vague promises, keeping buyers in the dark about how much they will actually get in the end (though such situations may arise in practice, see Gyselen, 2003).

[5] We could also consider schemes featuring more than one sales target, but we will see later that one threshold will generally be sufficient.

[6] Results are qualitatively similar if we restrict fixed payments to go from firms to buyers only, while ruling out franchise fees paid by buyers to firms.

[7] We will argue that rebate schemes, when applied to buyers who differ in size, will be a tool of (de facto) discrimination, even if the schemes as such are uniform. We will also discuss under what circumstances uniformity of contracts may be restrictive (in the sense that firms would want to specify individualized sales targets instead).

(i) ***Entry equilibrium:***

- if $\bar{s} \leq 1 - c_I$*, I sets* $p_I = c_I$*, E sets* $p_E = c_I - \varepsilon$*, and all buyers, after observing* $p_I - p_E > 0$*, buy from E.*

- if $\bar{s} > 1 - c_I$*, I sets* $p_I = p_I^*$*, E sets* $p_E = 1 - \bar{s}$*, and all buyers, after observing* $p_I - p_E > 0$*, buy from E.*

(ii) ***Miscoordination equilibrium:*** *I sets* $p_I = \tilde{p}_I \in [c_I, p_I^*]$ *(where* p_I^* *is firm I's monopoly price), E sets* $p_E = \min\{\tilde{p}_I, p_E^*\}$ *(where* p_E^* *is firm E's monopoly price), and all buyers, after observing* $p_I \leq \tilde{p}_I$ *and* $p_I - p_E \leq \tilde{p}_I$*, end up buying from I.*

Proof: see Appendix I

Let us now solve for the pure-strategy Nash equilibria of the game specified in Section 2. We will show that under rebates, the miscoordination equilibrium continues to exist for all parameter values, while the entry equilibrium does not exist for some parameter values.

3.1 The buyers' problem ($t = 1$)

When deciding which firm to buy from, buyers seek to maximize total surplus, which is the sum of net consumer surplus and possible lump-sum payments they receive from or have to pay to the firms. Define net consumer surplus as follows:

$$\begin{aligned} CS_i^j\left(p_i^j\right) &= \frac{1}{2}\left(1-p_i^j\right)q_i^j\left(p_i^j\right) && (5)\\ &= \begin{cases} \frac{1}{2}(1-K)\left(1-p_i^j\right)^2 & \text{if } s_i \geq \bar{s} \text{ and } p_i^j \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } j = l && (6)\\ &= \begin{cases} \frac{1}{2}\frac{K}{m}\left(1-p_i^j\right)^2 & \text{if } s_i \geq \bar{s} \text{ and } p_i^j \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } j = s && (7) \end{aligned}$$

Now, firm i's offer specifies a quantity threshold $\bar{q}_i$, a unit price for sales below and above this threshold,

$$p_i(q_i^j) = \begin{cases} p_{i,1} & \text{if } q_i^j < \bar{q}_i \\ p_{i,2} & \text{if } q_i^j \geq \bar{q}_i \end{cases}$$

where we allow for $p_{i,1} \neq p_{i,2}$, and a lump-sum payment $R_i(q_i^j)$

$$R_i(q_i^j) = \begin{cases} R_{i,1} & \text{if } q_i^j < \bar{q}_i \\ R_{i,2} & \text{if } q_i^j \geq \bar{q}_i \end{cases}$$

where we allow again for $R_{i,1} \neq R_{i,2}$. Note that $R_i(q_i^j) > 0$ denotes payments made by firm i to buyer j ("rewards"), while $R_i(q_i^j) < 0$ denotes payments made by buyer j to firm i ("franchise fees").

First, consider the large buyer $j = l$, and suppose his demand at price $p_{i,2}$ is above the threshold, i.e. $q_i^l(p_{i,2}) \geq \bar{q}_i$ for both firms $i = I, E$ (this will be the only relevant case). Then, the large buyer can either buy $q_i^l(p_{i,2})$ at price $p_{i,2}$ and receive (or pay) fixed payment $R_{i,2}$, or he can buy quantity $q_i^l = \min\left\{q_i^l(p_{i,1}), \bar{q}_i - \varepsilon\right\}$ at price $p_{i,1}$ and receive (or pay) fixed payment $R_{i,1}$.

If the large buyer buys below the threshold, i.e. if he buys $q_i^l = \min\left\{q_i^l(p_{i,1}), \bar{q}_i - \varepsilon\right\}$ at price $p_{i,1} < 1$, his net consumer surplus can be expressed as[8]

$$CS^{l,net}(p_{i,1}, \bar{q}_i) = \begin{cases} CS_i^l(p_{i,1}) & \text{if } s_i \geq \bar{s} \text{ and } q_i^l(p_{i,1}) < \bar{q}_i \\ \bar{q}_i\left(1 - p_{i,1} - \bar{q}_i \frac{1}{2}\frac{1}{1-K}\right) & \text{if } s_i \geq \bar{s} \text{ and } q_i^l(p_{i,1}) \geq \bar{q}_i \\ 0 & \text{otherwise} \end{cases}$$

The large buyer will want to buy from the incumbent iff

$$\begin{aligned} & \max\left\{CS_I^l(p_{I,2}) + R_{I,2}, CS^{l,net}(p_{I,1}, \bar{q}_I) + R_{I,1}\right\} \\ \geq\ & \max\left\{CS_E^l(p_{E,2}) + R_{E,2}, CS^{l,net}(p_{E,1}, \bar{q}_E) + R_{E,1}, 0\right\} \end{aligned}$$

Next, consider a typical small buyer $j = s$, and suppose his demand at price $p_{i,2}$ is below the threshold, i.e. $q_i^s(p_{i,2}) < \bar{q}_i$ for both firms $i = I, E$ (again, this will be the only relevant case). Then, a small buyer may either buy $q_i^s(p_{i,1})$ at price $p_{i,1}$ and pay (or receive) fixed payment $R_{i,1}$, or he may buy the sales target $\bar{q}_i$ at price $p_{i,2}$ in order to qualify for payment $R_{i,2}$.

Suppose the small buyer buys the sales target $\bar{q}_i$ at price $p_{i,2}$, a quantity that we assumed to be larger than his actual demand at price $p_{i,2}$, $q_i^s(p_{i,2}) < \bar{q}_i$.

Consider first the case where $\bar{q}_i \geq \frac{K}{m}$, i.e. the number of units a small buyer would have to buy to reach the sales target is (weakly) larger than the largest quantity he can consume, $q_i^s(p_i = 0) = \frac{K}{m}$. Suppose that the small buyer will consume all units he can consume, and that the excess units, $\bar{q}_i - \frac{K}{m}$, can be disposed of at no cost.[9] Then, the small buyer's gross consumer surplus is $\frac{1}{2}\frac{K}{m}$.

If instead $\bar{q}_i < \frac{K}{m}$, then the small buyer can consume all the units he buys, and his gross consumer surplus is $\bar{q}_i\left(1 - \frac{1}{2}\frac{m}{K}\bar{q}_i\right)$. Thus, a small buyer's net consumer surplus of buying $\bar{q}_i$ units can be defined as

$$CS^{s,net}(p_{i,2}, \bar{q}_i) = \begin{cases} \bar{q}_i\left(1 - p_{i,2} - \bar{q}_i \frac{1}{2}\frac{m}{K}\right) & \text{if } s_i \geq \bar{s} \text{ and } \bar{q}_i < \frac{K}{m} \\ \frac{1}{2}\frac{K}{m} - p_{i,2}\bar{q}_i & \text{if } s_i \geq \bar{s} \text{ and } \bar{q}_i \geq \frac{K}{m} \\ 0 & \text{otherwise} \end{cases}$$

The small buyer's net total surplus of buying $\bar{q}_i$ at price $p_{i,2}$ is the sum of net consumer surplus and fixed payment $R_{i,2}$, $CS^{s,net}(p_{i,2}, \bar{q}_i) + R_{i,2}$.

The small buyer will want to buy from the incumbent iff

$$\begin{aligned} & \max\left\{CS_I^s(p_{I,1}) + R_{I,1}, CS^{s,net}(p_{I,2}, \bar{q}_I) + R_{I,2}\right\} \\ \geq\ & \max\left\{CS_E^s(p_{E,1}) + R_{E,1}, CS^{s,net}(p_{E,2}, \bar{q}_E) + R_{E,2}, 0\right\} \end{aligned}$$

[8] where we simplify $\bar{q}_i - \varepsilon$ to $\bar{q}_i$

[9] Recall that we excluded reselling of units between buyers, so the only thing a small buyer can do with units he cannot consume is to throw them away.

Lemma 2 *(buyer-choice equilibrium) Let the firms' offers be given as* $\left\{\bar{q}_i, p_i(q_i^j), R_i(q_i^j)\right\}$ *for* $i = I, E$*. Let* I*'s offer satisfy the buyers' participation constraints (i.e. buyers prefer to buy from the incumbent rather than not buying at all):*

$$
\begin{aligned}
\max\left\{CS_I^s\left(p_{I,1}\right) + R_{I,1}, CS^{s,net}\left(p_{I,2}, \bar{q}_I\right) + R_{I,2}\right\} &\geq 0 \\
\max\left\{CS_I^l\left(p_{I,2}\right) + R_{I,2}, CS^{l,net}\left(p_{I,1}, \bar{q}_I\right) + R_{I,1}\right\} &\geq 0
\end{aligned}
$$

Then, there are two types of pure-strategy equilibria at the buyer-choice stage:

(i) all buyers buy from the incumbent; this equilibrium can arise under all offers; and

(ii) all buyers buy from the entrant; this equilibrium can only arise if offers are such that all buyers prefer to buy from the entrant over buying from the incumbent, and the entrant's offer is such that the total demand it generates is sufficient to reach the minimum size.

Proof: Since small buyers are identical, and offers must be uniform, the small buyers either all prefer to buy from I, or they all prefer to buy from E. Hence, offers can be such that

- all buyers prefer to buy from I over buying from E (or not buying at all). Then, all buyers buying from I is the only equilibrium.

- the large buyer prefers to buy from I, while all the small buyers prefer to buy from E. Then, the large buyer buys from I: The incumbent has an installed base that provides its network with the minimum size, so I can serve all buyers who want to buy from it, no matter how many (or how few) they are. Since $\bar{s} > \max\ \{1-K, K\}$, the demand generated by the small buyers alone is not sufficient for E to reach the minimum size. Recall that we ruled out side-payments among buyers, i.e. we do not allow the small buyers to offer some compensation to the large buyer to make him buy from E instead of I. Hence, the small buyers will be forced to buy from I as well (which they prefer over not buying at all). Again, all buyers buying from I is the only equilibrium.

- the large buyer prefers to buy from E, while all the small buyers prefer to buy from I. The same reasoning as before applies vice versa. Again, all buyers buying from I is the only equilibrium.

- all buyers prefer to buy from E. Now, there are two possible equilibria: either all buyers buy from E, or they all buy from I. If total demand under E's offer is not sufficient for E to reach the minimum size, all buyers buying from I is the only equilibrium. If instead the entrant's offer is such that the total demand it generates is sufficient to reach the minimum size, all buyers buying from E is obviously an equilibrium. But suppose all buyers buy from I instead. Then, $\bar{s} > \max\ \{1-K, K\}$ implies that none of the individual buyers alone is sufficient for E to reach the minimum size, and so no buyer will want to deviate and buy from E, even though E's offer is strictly better than I's offer. Thus, all buyers buying from I is another equilibrium in this case ("pure miscoordination" equilibrium). $\square$

3.2 The firms' problem ($t = 0$)

Let us now derive the incumbent's and entrant's equilibrium offers to the buyers, $\left\{\bar{q}_i, p_i(q_i^j), R_i(q_i^j)\right\}$ for $i = I, E$. Since $(p_{i,1}, R_{i,1})$ always refer to the small buyers, and $(p_{i,2}, R_{i,2})$ always refer to the large buyer, we will from now on denote $(p_{i,1}, R_{i,1})$ by $(p_i^s, R_{i,s})$, and $(p_{i,2}, R_{i,2})$ by $\left(p_i^l, R_{i,l}\right)$, for $i = I, E$.

We say that firm i's offer satisfies the *"sorting condition"* if the large buyer buys above the threshold, and the small buyers buy below the threshold, i.e. if

$$\begin{aligned} \max\left\{CS_i^l\left(p_i^l\right) + R_{i,l}, CS^{l,net}\left(p_i^s, \bar{q}_i\right) + R_{i,s}\right\} &= CS_i^l\left(p_i^l\right) + R_{i,l} \text{ and} \quad (8) \\ \max\left\{CS_i^s\left(p_i^s\right) + R_{i,s}, CS^{s,net}\left(p_i^l, \bar{q}_i\right) + R_{i,l}\right\} &= CS_i^s\left(p_i^s\right) + R_{i,s} \end{aligned}$$

Lemma 3 *(miscoordination equilibrium) For all parameter values, there is an equilibrium where I sets $p_I^s > c_I$, $p_I^l = c_I$, and fully extracts consumer surplus from the small buyers, while leaving some rent to the large buyer, E makes the analogous offer with $p_E^s \in (0, p_I^s)$, $p_E^l = 0$, and all buyers buy from I.*

Proof: see Appendix I

Let us now turn to the question when there is scope for E's entry. First, note that the maximum joint rent that I can generate when serving all buyers is

$$CS_I^l\left(c_I\right) + mCS_I^s\left(c_I\right)$$

which is strictly smaller than E's maximum joint rent,

$$CS_E^l\left(c_E\right) + mCS_E^s\left(c_E\right)$$

because $c_E < c_I$. Thus, whenever E's offer is such that buyers enjoy total net surplus of at least $CS_I^l\left(c_I\right) + mCS_I^s\left(c_I\right)$ when buying from E, it is impossible for I to match E's offer to *all* the buyers without making losses.

Recall, though, that in order to reach the minimum size, the entrant has to serve the large buyer plus at least one small buyer. Now, by Lemma 2, we have that whenever the large buyer prefers to buy from I, the small buyers will be forced to buy from I as well, even if they prefer to buy from E.

Thus, if E's offer is such that buyers enjoy total net surplus of $CS_I^l\left(c_I\right) + mCS_I^s\left(c_I\right)$, but not all of this rent is offered to the large buyer (some of it goes to the small buyers), it is sufficient for I to match E's offer to the large buyer (while leaving nothing to the small buyers) to prevent E's entry. Now, suppose I's offer reads

$$\begin{aligned} p_I^s &= p_I^l = c_I,\ \bar{q}_I = q_I^l\left(c_I\right) \\ R_{I,s} &= -CS_I^s\left(c_I\right),\ R_{I,l} = mCS_I^s\left(c_I\right) \end{aligned}$$

This is the best offer that I can make to the large buyer: I charges a unit price equal to marginal cost to all buyers, then fully extracts small buyers' net consumer surplus and transfers it all to the large buyer, who enjoys total net surplus of $CS_I^l(c_I) + mCS_I^s(c_I)$. [10]

In order to win the large buyer's orders, the entrant will have to match this offer, say by setting

$$p_E^l = c_E,\ R_{E,l} = CS_I^l(c_I) + mCS_I^s(c_I) - CS_E^l(c_E) + \varepsilon$$

Now, for this offer to be feasible, E has to make sure that the small buyers will want to buy from E as well. One may think that it would be sufficient for E to match I's offer to the small buyers, for instance by setting

$$p_E^s = p_E^l = c_E,\ \bar{q}_E = q_E^l(c_E),\ R_{E,s} = -CS_E^s(c_E) + \varepsilon$$

But this cannot be an entry equilibrium: recall that, by Lemma 2, whenever the small buyers strictly prefer to buy from I, the large buyer will be forced to buy from I as well, even if he prefers to buy from E. Then, to prevent E's entry, I can simply "turn the tables" and match E's offer to the small buyers, while extracting as much rent as possible from the large buyer (we will characterize I's best offer to the small buyers in the Proof of Proposition 4).

Hence, for E to successfully challenge I, E must be able to simultaneously match both I's best offer to the large buyer, and I's best offer to the small buyers. Then, even though E is more efficient than I, I may use rebates so as to prevent E's entry.

Proposition 4 *(entry equilibria)*

(i) If E's joint net surplus is not sufficient to match both I's best offer to the large buyer and I's best offer to the small buyers simultaneously, then no "entry equilibrium" exists, so that our game only has a "miscoordination equilibrium". This is more likely to happen the smaller the efficiency gap between I and E, and the more fragmented buyers are.

(ii) If

- E's joint net surplus is sufficient to match both I's best offer to the large buyer and I's best offer to the small buyers simultaneously, and

- E's offer satisfies the "sorting condition" of equation (8),

then our game has two pure-strategy equilibria: one where all buyers buy from E ("entry equilibrium"), and one where all buyers buy from I ("miscoordination equilibrium").

Proof: see Appendix I

[10] Note that under this offer, buyers prefer to buy from I rather than not buying at all. Assume for now that this offer also satisfies the "sorting condition", in particular that the small buyers prefer to buy $q_I^s(c_I)$ and pay $R_{I,s} = -CS_I^s(c_I)$ rather than buying $\bar{q}_I$ in order to qualify for $R_{I,l} = mCS_I^s(c_I)$. We will deal with this issue in the Proof of Proposition 4.

4 Concluding remarks

The purpose of this exercise was to demonstrate the exclusionary potential of rebate arrangements in the presence of network externalities. We have shown that rebates may allow the incumbent to prevent entry in cases where that would not have been possible under uniform flat prices. The finding is particular interesting insofar as, in our model, the entrant is in a fairly good initial position compared to other papers on exclusionary practices: it does not have to pay any fixed cost to start operating in the industry, entrant and incumbent can approach buyers simultaneously (i.e. the incumbent has no first-mover advantage in offering contracts to the buyers before the entrant can do so), and the entrant has the same contractual instruments at its disposal. Yet, as it turns out, if rebates (even if uniform) are paired with network externalities, and the incumbent disposes of the advantage of a mature network, rebates turn out to work in favor of the incumbent, even though the entrant is more efficient.

Our analysis is very preliminary and should be further developed in the following directions:

- What if we allow for open discrimination among buyers?
- What if buyers can coordinate their actions among each other?
- What if buyers are allowed to patronize more than one firm?
- Is our model robust to the large buyer being sufficient for the entrant to reach the minimum size?

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5 Appendix I

Proof of Proposition 1:

(i) Let $\bar{s} \leq 1 - c_I$, so that $p_E = c_I - \varepsilon$. Then, with all buyers buying from E, total demand is $mq_E^s(p_E) + q_E^l(p_E) = 1 - p_E > \bar{s}$, and so E will reach (and even exceed) the minimum size. No buyer will want to deviate and buy from I: I's product has the same value as E's product, but sells at a strictly higher price. I will not want to deviate either: if I lowers its price to match E's offer, I will attract all buyers, but will sell at a loss ($p_E < c_I$); and increasing p_I above c_I will not attract any buyers. E has no incentive to change anything about its price either: increasing p_E would imply losing the buyers to I, and decreasing p_E will just reduce profits (recall that E is not radically more efficient than I, i.e. $p_E = c_I - \varepsilon < p_E^*$, so that E cannot gain from reducing its price below $c_I - \varepsilon$).

Let $\bar{s} > 1 - c_I$, and suppose E sets $p_E = 1 - \bar{s}$. Then, total demand generated by all buyers is $mq_E^s(p_E) + q_E^l(p_E) = 1 - p_E = \bar{s}$, and so E will just reach the minimum size, while still breaking even ($\bar{s} \leq 1$ and $c_E = 0$ imply $p_E - c_E \geq 0$). E has no incentive to increase its price, as that would imply falling short of the minimum size, while charging a price below $1 - \bar{s}$ would only reduce profits . As before, no buyer will want to deviate and buy from I. If I decreases its price to a value $p_I \in (1 - \bar{s}, p_I^*)$ or increases it to some $p_I > p_I^*$, I will not be able to attract any buyers, and selling at a price at or below $1 - \bar{s}$ (which is strictly smaller than c_I if $\bar{s} > 1 - c_I$) would imply losses.

Note that we eliminate all equilibria in weakly dominated strategies, where

- if $\bar{s} \leq 1 - c_I$, I sets $p_I \in (0, c_I)$ instead of $p_I = c_I$, and E sets $p_E = p_I - \varepsilon$, and
- if $\bar{s} > 1 - c_I$, I sets $p_I \in (1 - \bar{s}, p_I^*)$ or $p_I > p_I^*$, and E sets $p_E = 1 - \bar{s}$.

(ii) Suppose that all buyers buy from I. Then, recall that $\bar{s} > \max\,\{1 - K, K\}$, implying that none of the individual buyers alone is sufficient for E to reach the minimum size. Thus, E's product has zero value for any single buyer, and so no buyer will want to deviate and buy from E, even though E's price is (weakly) below I's price. I sets $p_I = \tilde{p}_I$, which is the highest possible $p_I \in [c_I, p_I^*]$ such that any price above $\tilde{p}_I$ would be followed by a continuation equilibrium where all buyers would switch to E (since $\tilde{p}_I + \varepsilon > p_E = \min\{\tilde{p}_I, p_E^*\}$). Thus, I has no incentive to increase or decrease its price. Since buyers will not switch to E even if the price difference between the two firms is maximal, i.e. even if E charges $p_E = 0$, E has no incentive to decrease its price.

We eliminate all equilibria in weakly dominated strategies, where

- if $\tilde{p}_I < p_E^*$, E sets $p_E < \tilde{p}_I$, or
- if $\tilde{p}_I \geq p_E^*$, E sets $p_E < p_E^*$ or $p_E \in (p_E^*, \tilde{p}_I]$.$\square$

Proof of Lemma 3:

Recall from Lemma 2 that buyers will buy from the incumbent if all or some of them (i.e. either the large buyer, or the small buyers) prefer to do so, and may buy from the incumbent if none of them prefers to do so. Now, suppose there is an equilibrium of the whole game where all buyers buy from I, and the small buyers actually prefer to do so (the reasoning is analogous if instead the large buyer, or all buyers, prefer to buy from I). Then, I could reduce the small buyers' net total surplus even below E's offer without losing them to E (given that the large buyer still buys from I). This would be a profitable deviation for I, since increasing p_I^s or reducing $R_{I,s}$ increases I's profits.

Hence, any equilibrium where all buyers buy from I must be a pure miscoordination equilibrium, i.e. all buyers buy from I although they would prefer to buy from E. Consider such a candidate equilibrium. Suppose that I wants to extract the full rent from all buyers, setting a uniform unit price of $p_I(q_I^j) = c_I$ for all $q_I^j \geq 0$, and franchise fees

$$R_I(q_I^j) = \begin{cases} -CS_I^s(c_I) & \text{if } q_I^j \leq \bar{q}_I = q_I^s(c_I) \\ -CS_I^l(c_I) & \text{if } q_I^j > \bar{q}_I = q_I^s(c_I) \end{cases}$$

This "first-best" offer (from the point of view of I) is not feasible, because the large buyer would then prefer to buy below the threshold, i.e. buying $\bar{q}_I = q_I^s(c_I) < q_I^l(c_I)$, and enjoying strictly positive net surplus

$$CS^{l,net}(c_I, q_I^s(c_I)) - CS_I^s(c_I) > 0$$

Thus, non-discrimination implies that I's offer to the large buyer must leave the latter with at least as much rent as buying below the threshold would yield.

I has to solve the following profit maximization problem:

$$\max_{\{p_I^s, p_I^l, R_{I,s}, R_{I,l}\}} m(p_I^s - c_I)\, q_I^s(p_I^s) + m(-R_{I,s}) + \left(p_I^l - c_I\right) q_I^l\left(p_I^l\right) + (-R_{I,l})$$

subject to the large buyer's incentive constraint and the small buyers' participation constraint:

$$\begin{aligned} CS_I^l\left(p_I^l\right) + R_{I,l} &\geq CS^{l,net}(p_I^s, q_I^s(p_I^s)) + R_{I,s} \\ CS_I^s(p_I^s) + R_{I,s} &\geq 0 \end{aligned}$$

Now, any solution to this problem requires that the two constraints hold with equality, thus determining $R_{I,l}$ and $R_{I,s}$, and that $p_I^l = c_I$ (which, given p_I^s, maximizes the rent that I can extract from the large buyer).

Therefore, the incumbent's problem reduces to choosing the right p_I^s that balances the following trade-off:

- a higher p_I^s will imply a deadweight loss on sales to the small buyers, which reduces small buyers' consumer surplus and thus the maximum $R_{I,s}$ that I can charge.

- a higher p_I^s will reduce the large buyer's payoff from mimicking the small buyers, thus allowing I to charge a higher $R_{I,l}$.

The resulting maximization problem is convex in p_I^s, and solves for

$$p_I^{s,opt} = \frac{c_I + \frac{1}{m}\left(1 - \frac{1}{1-K}\frac{K}{m}\right)}{1 + \frac{1}{m}\left(1 - \frac{1}{1-K}\frac{K}{m}\right)}$$

Our assumption that $c_I < 1$ implies $p_I^{s,opt} \in (c_I, 1)$. The large buyer's participation constraint holds by construction ($CS^{l,net}(p_I^s, q_I^s(p_I^s)) > CS_I^s(p_I^s)$ for all p_I^s and $CS_I^s(p_I^{s,opt}) = -R_{I,s}^{opt}$ imply $CS_I^l(c_I) + R_{I,l}^{opt} > 0$).

The small buyers' sorting condition holds as well: Suppose a small buyer considers buying the large buyer's threshold, $\bar{q}_{I,l} = q_I^l(c_I)$, at price c_I. Then, this buyer would enjoy net consumer surplus $CS_I^s(c_I)$ from consuming the first $q_I^s(c_I)$ units, and negative surplus on all the remaining units $\bar{q}_{I,l} - q_I^s(c_I)$ (which have to be bought at price c_I, but have a value less than c_I to the small buyer). Thus, the small buyer's net consumer surplus from consuming $\bar{q}_{I,l} = q_I^l(c_I)$ will be strictly lower than $CS_I^s(c_I)$. However, the small buyer will have to pay $R_{I,l}^{opt}$ (instead of $R_{I,s}^{opt} = -CS_I^s(p_I^{s,opt})$), where $R_{I,l}^{opt} < -CS_I^s(c_I)$. Thus, a small buyer buying the large buyer's threshold would end up with a strictly negative net total surplus, and so he will prefer to buy the small buyers' threshold $\bar{q}_{I,s} = q_I^s(p_I^{s,opt})$.

Hence, the incumbent's equilibrium offer will be

$$\begin{aligned} p_I^s &= p_I^{s,opt} > c_I,\ p_I^l = c_I,\ \bar{q}_{I,s} = q_I^s\left(p_I^{s,opt}\right),\ \bar{q}_{I,l} = q_I^l(c_I) \\ R_{I,s} &= -CS_I^s\left(p_I^{s,opt}\right),\ R_{I,l} = CS^{l,net}\left(p_I^{s,opt}, q_I^s\left(p_I^{s,opt}\right)\right) + R_{I,s} - CS_I^l(c_I) \end{aligned}$$

The entrant is indifferent about the offer it makes (given that our candidate equilibrium is a pure miscoordination equilibrium). Eliminating all equilibria where E plays weakly dominated strategies, E will just solve the analogous optimization problem analyzed above, which yields

$$\begin{aligned} p_E^{s,opt} &= \frac{\frac{1}{m}\left(1 - \frac{1}{1-K}\frac{K}{m}\right)}{1 + \frac{1}{m}\left(1 - \frac{1}{1-K}\frac{K}{m}\right)} < p_I^{s,opt} \\ p_E^s &= p_E^{s,opt} > 0,\ p_E^l = c_E < p_I^l,\ \bar{q}_{E,s} = q_E^s\left(p_E^{s,opt}\right),\ \bar{q}_{E,l} = q_E^l(c_E) \\ R_{E,s} &= -CS_E^s\left(p_E^{s,opt}\right),\ R_{E,l} = CS^{l,net}\left(p_E^{s,opt}, q_E^s\left(p_E^{s,opt}\right)\right) + R_{E,s} - CS_E^l(c_E) \end{aligned}$$

But given that all buyers buy from I, no individual buyer will want to deviate and buy from E, and so all buyers will end up buying from I, even though E offers more total net surplus to each buyer than I does.□

Proof of Proposition 4:

(i) We argued in Lemma 3 that miscoordination equilibria exist for all parameter values. Thus, if no entry equilibrium exists, then the miscoordination equilibrium is the only pure-strategy equilibrium of our game.

Now, recall from Lemma 2 that at the buyer choice stage, buyers will buy from the entrant iff all buyers prefer to buy from the entrant over buying from the incumbent. Thus, a necessary condition for existence of an entry equilibrium is that I can neither match E's offer to the large buyer nor E's offer to the small buyers.

The best offer that I can make to the small buyers solves

$$\max_{\{p_I^s, p_I^l, R_{I,s}, R_{I,l}\}} CS_I^s(p_I^s) + R_{I,s}$$

subject to the large buyer's sorting condition and the incumbent's break-even constraint:

$$\begin{aligned} CS_I^l\left(p_I^l\right)+R_{I,l} &\geq CS^{l,net}\left(p_I^s,\bar{q}_I\right)+R_{I,s} \\ mR_{I,s}+R_{I,l} &\leq m\left(p_I^s-c_I\right)q_I^s\left(p_I^s\right)+\left(p_I^l-c_I\right)q_I^l\left(p_I^l\right) \end{aligned}$$

Now, any solution to this problem requires that the two constraints hold with equality, thus determining $R_{I,l}$ and $R_{I,s}$, and that $p_I^l = c_I$ (which, given p_I^s, maximizes the rent that I can extract from the large buyer).

Thus, I's problem reduces to choosing the right p_I^s that balances the following trade-off:

- a higher p_I^s will imply a deadweight loss on sales to the small buyers, which reduces small buyers' consumer surplus;

- a higher p_I^s will reduce the large buyer's payoff from buying below the threshold, thus allowing I to charge a higher $R_{I,l}$, and hence to transfer more rent to the small buyers without violating the sorting condition.

The problem is convex in p_I^s and solves for

$$p_I^{s,opt}=\frac{\frac{1}{m+1}\left(1+c_I+\frac{2}{m}\left(1-\frac{1}{2}\frac{1}{1-K}\frac{K}{m}\right)\right)-\frac{1}{m}}{\frac{1}{m+1}\left(2+\frac{2}{m}\left(1-\frac{1}{2}\frac{1}{1-K}\frac{K}{m}\right)\right)-\frac{1}{m}}$$

where $p_I^{s,opt} \in (c_I,1)$. By construction, our solution also satisfies the small buyers' sorting condition, as well as the participation constraints

$$CS_I^s\left(p_I^{s,opt}\right)+R_{I,s}^{opt}\geq 0 \text{ and } CS_I^l\left(c_I\right)+R_{I,l}^{opt}\geq 0.$$

The incumbent's "first-best" offer to the large buyer (from the point of view of the large buyer) is the one that allows I to extract the full net consumer surplus from the small buyers without violating their sorting condition:

$$\begin{aligned} p_I^s &= p_I^l=c_I,\ \bar{q}_I=q_I^l\left(c_I\right) \\ R_{I,s} &= -CS_I^s\left(c_I\right),\ R_{I,l}=mCS_I^s\left(c_I\right) \end{aligned}$$

with the small buyers' sorting condition being

$$0\geq CS^{s,net}\left(p_I^l,\bar{q}_I\right)+=R_{I,l}\underbrace{m\frac{1}{2}\frac{K}{m}\left(1-c_I\right)^2}$$

which can be rearranged to

$$K\leq\frac{c_I\left(1-c_I\right)}{\frac{1}{2}\frac{1}{m}+c_I\left(1-c_I\right)+\frac{1}{2}\left(1-c_I\right)^2}=K^*\left(c_I,m\right) \tag{9}$$

If the small buyers' sorting condition is violated, then I has to solve the following program:

$$\max_{\left\{p_I^s,p_I^l,R_{I,s},R_{I,l}\right\}} CS_I^l\left(p_I^l\right)+R_{I,l}$$

subject to the small buyers' sorting condition and the incumbent's break-even constraint:

$$\begin{aligned} CS_I^s(p_I^s) + R_{I,s} &\geq CS^{s,net}\left(p_I^l, \bar{q}_I\right) + R_{I,l} \\ mR_{I,s} + R_{I,l} &\leq m\left(p_I^s - c_I\right) q_I^s\left(p_I^s\right) + \left(p_I^l - c_I\right) q_I^l\left(p_I^l\right) \end{aligned}$$

Now, any solution to this problem requires that the two constraints hold with equality, thus determining $R_{I,l}$ and $R_{I,s}$, and that $p_I^s = c_I$ (which, given p_I^l, maximizes the rent that I can extract from the small buyers).

If $K \leq \frac{m+1-c_I}{m+2+\frac{1}{m}-c_I}$, the problem is convex in p_I^l and solves for:

$$p_{I,1}^{l,opt} = \frac{1}{m+1} c_I$$

If $K > \frac{m+1-c_I}{m+2+\frac{1}{m}-c_I}$, we obtain

$$p_{I,2}^{l,opt} = \frac{\frac{m}{K}(1-K) - 1 - \frac{c_I}{m}}{\frac{m}{K}(1-K) - 1 - \frac{1}{m}}$$

with the second-order condition being satisfied iff $\frac{m}{K}(1-K) - 1 - \frac{1}{m} < 0$. In either case, $p_I^{l,opt} < c_I$, i.e. I sells to the large buyer below marginal cost, and the large buyer's total net surplus will be lower than under the "first best" offer, where it is $CS_I^l(c_I) + mCS_I^s(c_I)$.

Now, if the total net surplus that E can generate, $CS_E^l(c_E) + mCS_E^s(c_E)$, is smaller than the sum of I's best offer to the large buyer and I's best offer to the small buyers, then I can always match either E's offer to the large buyer, or E's offer to the small buyers, implying that there is no entry equilibrium in pure strategies.

The corresponding resource conditions read:

- if $K \leq K^*(c_I, m)$:

$$CS_E^l(c_E) + mCS_E^s(c_E) < CS_I^l(c_I) + mCS_I^s(c_I) + m\left(CS_I^s\left(p_I^{s,opt}\right) + R_{I,s}\left(p_I^{s,opt}\right)\right)$$

-if $K > K^*(c_I, m)$:

$$CS_E^l(c_E) + mCS_E^s(c_E) < CS_I^l\left(p_I^{l,opt}\right) + R_{I,l}\left(p_I^{l,opt}\right) + m\left(CS_I^s\left(p_I^{s,opt}\right) + R_{I,s}\left(p_I^{s,opt}\right)\right)$$

Let $K \leq K^*(c_I, m)$. If the efficiency gap between I and E becomes very small, i.e. if $c_I \to 0$, then

$$CS_E^l(c_E) + mCS_E^s(c_E) - CS_I^l(c_I) + mCS_I^s(c_I) \to 0$$

and so the resource condition reduces to $0 < m\left(CS_I^s\left(p_I^{s,opt}\right) + R_{I,s}\left(p_I^{s,opt}\right)\right)$, which is always satisfied. Thus, the smaller the efficiency gap between I and E, the more likely it is that I can prevent E's entry.

The analogous reasoning applies if $K > K^*(c_I, m)$: as $c_I \to 0$, we have that $\frac{m+1-c_I}{m+2+\frac{1}{m}-c_I} \to \frac{1}{1+\frac{1}{m}}$, which is the upper bound on K deriving from $1 - K > \frac{K}{m}$. Thus, we can only have $K \leq \frac{1}{1+\frac{1}{m}}$, and so I will set $p_{I,1}^{l,opt} = \frac{1}{m+1} c_I \to 0$. Thus,

$$CS_E^l(c_E) + mCS_E^s(c_E) - CS_I^l\left(p_I^{l,opt}\right) + R_{I,l}\left(p_I^{l,opt}\right) \to 0$$

and we have, as before, the resource condition reducing to $0 < m\left(CS_I^s\left(p_I^{s,opt}\right) + R_{I,s}\left(p_I^{s,opt}\right)\right)$.

Note also that the more fragmented the buyers are, in particular the higher the number of small buyers m, the more likely it is that $K \leq K^*(c_I, m)$, allowing I to extract the full small buyer consumer surplus and transfer it to the large buyer, which makes it more difficult for the entrant to match both of I's best offers.

(ii) TO BE DONE.$\square$